Grade/Subject	Grade 8/ Mathematics
Unit Title	Unit 8: Patterns and Data
Overview of Unit	Investigate patterns of association in bivariate data.
Pacing	

Background Information For The Teacher
Students are introduced to statistics in grade 6, where they developed an understanding of statistical variability and summarized and described distributions. (6.SP) In grade 7, students used random sampling to draw inferences about a population, drew informal comparative inferences about two populations, investigated chance processes and developed, used, and evaluated probability models. (7.SP) Until Grade 8, almost all of students' statistical topics and investigations have dealt with univariate data (i.e. Collections of counts or measurements of one characteristic). This unit will introduce the students to bivariate data and the association that can exist between them. *Until CCSS is fully implemented in prior grades, the suggested pacing guide may need to be modified to meet the needs of students who have not had exposure to the content in grades 6 and 7.

Essential Questions (and Corresponding Big Ideas)
Why is data collected and analyzed? • Data collection and analysis assigns meaning to the information and allows you to understand the implications. How do people use data to influence others? • Data can be used to influence others by drawing conclusions from and recognizing the significance of the information

analyzed.

How can predictions be made based on data?

- Data analysis allows you to draw conclusions and make predictions based on possible association between the two variables.

Core Content Standards	Explanations and Examples
8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. Students study scatter plots of bivariate data by constructing and interpreting them in terms of patterns they can see. They look for the patterns of clustering, outliers, positive or negative association, and linear or non-linear association. Examples of scatter plots below show positive and negative associations, clustering, and an outlier. <u>What the Teacher Does:</u> • Provide students with many examples of scatter plots showing bivariate data to help them understand what bivariate data are versus univariate data. Encourage students to collect their own bivariate data on a topic of interest to them. These activities can range from suggestions by individual students to group projects involving the collection and organization of the data into a scatter plot. This helps students clarify their understanding of what constitutes bivariate data.	8.SP.1. Students build on their previous knowledge of scatter plots examine relationships between variables. They analyze scatter plots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets. (http://nces.ed.gov/nceskids/createagraph/default.aspx) Examples: • Data for 10 students' Math and Science scores are provided in the chart. Describe the association between the Math and Science scores. • Data for 10 students' Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school. • Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Science	68	70	83	33	60	27	74	63	40	96

Student	1	2	3	4	5	6	7	8	9	10
Math score	64	50	85	34	56	24	72	63	42	93
Dist from school (miles)	0.5	1.8	1	2.3	3.4	0.2	2.5	1.6	0.8	2.5

- Use several different scatter plots that show clustering; outliers; positive, negative or no association; and linear or nonlinear associations. One approach is to let students sort the different scatter plots by what they see (i.e., positive or negative association patterns, linear or nonlinear associations, graphs with outliers). The scatter plots should be a mix of those that have no labels and those that do. This adds to the meaning of the different associations. Fro example, a scatter plot that has no association may have points scattered around, but that association brings meaning when the data are identified as the number of hours eighth graders sleep versus the number of hours they spend playing sports.

- Plan for students to explain the patterns in writing and orally using the correct terminology.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Students focus on linear patterns of association in scatter plots and understand that linear models (straight lines) are commonly used to model linear relationships. Then they begin to informally fit a straight line to the data and learn to assess its fit by judging the closeness of the line to the data points. The most appropriate line is the one that comes closest to most data points. The use of linear regression is not expected at this grade.

Number of staff	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

- The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

Date	1970	1975	1980	1985	1990	1995	2000	2005
Life Expectancy (in years)	70.8	72.6	73.7	74.7	75.4	75.8	76.8	77.4

What the Students Do:

- Model bivariate data in a scatter plot showing the different types of associations.
- Describe orally and/or in writing different patterns of association when presented with scatter plots of bivariate data. Explain what the different patterns cm=mean in specific contexts

Misconceptions and Common Errors:

Sometimes when a scatter plot shows no association, students are confused. Give them examples of data that my have no association to provide a context for reasoning why there would be no association. An example is the length of a person's hair and his or her final grade in mathematics.

8.SP.2 Examples:

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

Miles Traveled	0	75	120	160	250	300
Gallons Used	0	2.3	4.5	5.7	9.7	10.7

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What the Teacher Does:

• Facilitate a discussion about straight lines being widely used to model relationships between two quantitative variables in the real world. Ask students questions to involve them in the discussion such as, “Do you know of any situations where linear modes are used? What do you remember about the lines we have been graphing in class? Name some examples from class. Why do you think linear models are used so often?”
• Pose problems for students to solve using linear models. Encourage students to make predictions based on their lines.
• Display a few scatter plots that show a linear association. Ask students to informally fit a straight line. Students should informally draw a line of fit for a scatter plot and informal measure the strength of fit. Discussions should include questions such as, “What does it mean if a point is above the line? Below the line? Students should interpret the slope and intercept of the line of fit in the context of the data.

8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

Students practice solving contextual linear problems. The problems involve situations using bivariate measurement data such as those collected in a biology experiment. This standard connects with what students have learned about models of linear equations, slope, and intercept.

What the Teacher Does:

• Present students with measurement data on two variables and ask them to use a linear model to answer questions that involve interpreting slope and intercept. Ask the students to
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What the Students Do:
• Understand that straight lines are widely used to model relationships between two quantitative variables in the real world and can name some examples.
• Model real-world linear relationships on a graph.
• Construct straight lines to fit data presented in scatter plots, informally.
• Justify a fit line as a good fit (or not).
• Explain orally and/or in writing, the meaning of the fit line and its properties in terms of the context of the graph.

Misconceptions and Common Errors:
A common misconception students have is that a line of fit must go through at least some of the data points on the scatter plot. Expose students to examples where a line of fit goes through all of the data points and those where it does not go through any but both lines are good fits.

8.SP.3. Examples:
1. Given data from students’ math scores and absences, make a scatter plot.
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create a scatter plot, informally fit a line, and use the line to create a linear equation. Students will need the slope and intercept of the line to create the equation. Then students can answer questions or make predictions about the data. For the example, questions could be, "What could we expect with 3 more hours of sunlight each day? How much sunlight could we continue to add before the graph is no longer useful or the situation?"

- Pose problems like this example:

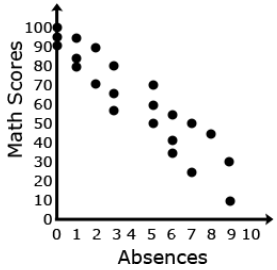
"In a mathematics class, the teacher kept a record of absences and grades.

Show these data on a scatter plot and fit a line to the data. Determine an approximate linear equation for this line. What does the slope of this line say about the relationship?"

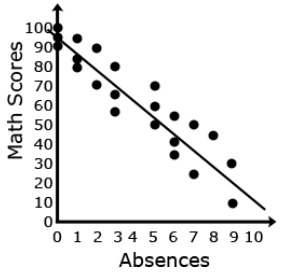
Absences	Grades
2	87
4	75
0	99
1	95
1	92
3	65
2	89
5	50
2	85
2	90
3	80

8.SP.4. Understand that patterns of association can also be seen in bivariate

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80



2. Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.



3. From the line of best fit, determine an approximate linear equation that models the given data (about $y = -\frac{25}{3}x + 95$)

4. Students should recognize that 95 represents the y-intercept and $-\frac{25}{3}$ represents the slope of the line.

5. Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4

categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew.*

Standard 4 asks students to switch from using numerical data to categorical data and use frequencies to answer questions about possible associations (linear/nonlinear, positive/negative/no association). Students construct and interpret tables that display categorical data on two different variables from the same subjects. A two-way table is a table that shows categorical data classified in two different ways. An example of a two way table that records possible data from the example in the standard about chores and curfews may be the following:

CURFEW

CHORES	YES	NO
YES	44	20
NO	20	44

One Interpretation of the chart is that of the students who answered yes, they had a curfew, 44 had chores and 20 did not. Of the students who answered no, they did not have a curfew, 20 had chores and 44 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.

What the Teacher Does:

- Present information on two categorical variables in a two-way table such as the example in the standard. Facilitate a

absences should expect to receive a math score of about 62. They can then compare this value to their line.

What the Students Do:

- Solve problems using a linear equation to model bivariate measurement data in context.
- Fit a line to the data, interpret the slope and intercept for the context, write the linear equation, and make predictions from the line.

Misconceptions and Common Errors:

There can be many steps before a student is ready to answer the specific questions asked in these problems. For struggling learners, scaffold the questions until they can make sense of the steps themselves.

8.SP.4. Example:

• The table illustrates the results when 100 students were asked the survey questions: Do you have a curfew? and Do you have assigned chores? Is there evidence that those who have a curfew also tend to have chores?

Curfew

Chores		Yes	No
	Yes	40	10
	No	10	40

discussion that leads students to conclude that categorical data can also be described numerically through the use of a two-way table. • Pose the question, "Do you see a pattern in the responses to indicate an association in the data? What type of association?" • Provide many opportunities for students to interpret categorical data presented in two-way tables. • Give students time to collect categorical data, create their own two-way table, and justify and patterns of association they find.	Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores. <u>What the Students Do:</u> • Understand the use of a two-way table to display bivariate categorical data. • Collect categorical data on two variables from the same group of people, display them in a two-way table, and interpret the data for associations. Justify orally or in writing the association using precise mathematical language. <u>Misconceptions and Common Errors:</u>

	As students collect their own data, be aware of them asking the same two questions of the same people. Some students may ask each person only one question. Students may have to use tally marks for frequency while collecting the data. Many students will be content with asking a few friends. Students should understand they need a significant sample size to find an association and the sample should be random.
Standards for Mathematical Practice	**Explanations and Examples**
Investigate patterns of association in bivariate data. **8.SP.1, 8.SP.2, 8.SP.3, and 8.SP.4** Grade 8 focuses on bivariate data and the patterns of association such as clustering, outliers, positive or negative association, and linear or nonlinear association. Students informally assess a model fit for a line and solve problems involving bivariate data, slope and intercepts. Students also work with bivariate categorical data using relative frequency. **MP1. Make sense of problems and persevere in solving them.** **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.** **MP6. Attend to precision.** **MP.7 Look for and make use of structure.** **MP.8 Look for and express regularity in repeated reasoning.**	Students solve problems using a linear model in the context of bivariate data. Students informally assess a line of best fit to data. Students use a linear model to solve problems. Students solve problems efficiently, accurately, and with the degree of precision appropriate for the context of the problem. Students look for structure in word problems to find linear patterns. Students understand the broader application of patterns in bivariate data and see the structure in similar situations.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*
Scatter Plots Patterns o Clustering o Outliers o Positive or Negative o Linear o Nonlinear Frequencies Two-way table o variables Equation of Linear Model o Slope and Intercept The line of best fit represents the data set as a whole, fitted through the majority of points. The characteristics of a linear relationship, such as: -Can be written in the form y=mx + b in which x is the independent variable, y is the dependent variable, m is the slope, and b is the y-intercept. -Appears to be a straight line in a xy coordinate graph. -When the constant rate is positive, the line will extend northeast and southwest. -When the constant rate is negative, the line will extend northwest and southeast.	• Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. • Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. • Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, informally assess the model fit by judging the closeness of the data points to the line, and make sense of the slope and y-intercept. • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* • Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

Strategies for modeling a line of best fit given a set of data. -Line drawn with the least total deviation from the actual data points. -Strategies include using a graphing calculator, strand of spaghetti, ruler, etc… Linear trends can be identified as positive or negative, while some trends have no correlation. Identifying outliers and other data characteristics allows for meaningful data interpretation and analysis.	• Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. • Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

UNDERSTAND

Big ideas, generalizations, principles, concepts, ideas that transfer across situations

The same characteristics used to describe linear relationships allow us to describe, classify, and analyze the association of bivariate measurement data.

Common Student Misconceptions for this Unit

Students may:

- Confuse the two axes of a graph.
- Not understand the meaning of points in the same position relative to one of the axes
- Think that the points on the graph stay in the same position even if the axes change
- Think that graphs are “pictures” of situations, rather than abstract representations
  Examples:
  Thinking that a speed graph of a bicycle coasting downhill and then uphill resembles the hill, first going down and then up;
  Thinking that a graph with negative slope means the object is falling;
  If the graph is rising, the object is moving upward; or if the graph changes direction, the object changes direction; if two lines on a graph cross, the paths of the objects cross.
- Think that graphs always go through the origin or always begin at the origin.

- Think that graphs always cross both axes
- Focus on some attributes of a situation and ignore others
  Example:
  Note the existence of local minima but ignore their relative positions or values.
- Read the y-axis as speed even when it represents a different parameter.
- Think that the greatest numbers labeled on the axes represent the greatest values reached.
  Example:
  If a graph of a race has the distance axis labeled up to 120 meters, the race is for 120 meters (even if it is a 100-meter race).
- Think that all sequences are linear or increasing and linear.
- Not discriminate between linear and non-linear sequences.

Unit Assessment/Performance Task	DOK
Unit 8 Test Unit 8 Performance Task "Class Attendance" Unit 8 Performance Task "Questions vs. Test Scores	

Vocabulary
• Bivariate data • Categorical data • Cluster • Distribution • Five-number summary • Frequency • Histogram

- Line of best fit
- Mean absolute deviation
- Negative Association
- Outlier
- Positive Association
- Qualitative Data
- Quantitative Variable
- Relative Frequency
- Scatter plot
- Slope
- Standard deviation
- Statistics
- Statistical question
- Symmetric
- Two-Way Table
- Univariate
- Variability
- Y-intercept

Key Learning Activities/Possible Lesson Focuses (order may vary)

These are ideas for lessons.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Lesson 1 Skills - Scatter Plots – 8.SP.1

- How to construct a scatter plot
  - From a table
- How to interpret a scatter plot
  - Considering different associations and patterns

Lesson 2 Skills – Lines of Best Fit – 8.SP.2

- How to informally estimate a line of best fit
  - slope-intercept form estimation connection
- How to use line of best fit to make a conjecture

Lesson 3 Skills – Two-Way Tables - 8.SP.4

- Construct a two-way table
  - Venn diagram as a foundation introduction
- Interpret and summarize data from a two-way table
- Find and Interpret Relative Frequencies from two-way tables

Supplemental Materials and Resources

http://new.censusatschool.org.nz/wp-content/uploads/2012/11/handout.pdf
This site offers several data sources for statistical information plus some ideas about how to make statistics for the 21st century.

http://www.nsa.gov/academia/_files/collected_learning/middle_school/interdisciplinary/scatter_plot.pdf
Several lessons creating scatter plots. Positive, negative and no relationships on scatter plots with questions for students to analyze outcomes.

http://illuminations.nctm.org/activitydetail.aspx?ID=146
This activity allows the user to enter a set of data, plot the data on a coordinate grid, and determine the

equation for a line of best fit. (Interactive)

http://illuminations.nctm.org/LessonDetail.aspx?ID=L571
In this lesson, students will compare the price of a toll to the distance traveled. Students will investigate data numerically and graphically to determine the per mile charge, and they will predict the cost if a new toll booth were added along the route.

http://map.mathshell.org/materials/tasks.php?taskid=381&subpage=apprentice
In this task, students will use a scatter diagram to compare the results of two school tests

Literature connections:
Data Analysis: An Introduction - Michael Lewis-Beck

Introduction to Statistics and Data Analysis - Powell's Books

Functions and Graphs (Dover Books on Mathematics) [Paperback] I. M. Gelfand

Interdisciplinary connections:
Language Arts
- Writing strategies to generate ideas analyze and predict results of scatter plots.

Social Studies
- Graphing data of elections, historical events and activities.

Science
- Data from science experiments can be graphed on a scatter plot

Tools/Manipulatives

Graphing calculators
Rulers
Graph paper
Board to use for incline
3 spheres of different known masses

Stopwatches Microsoft Excel (or comparable computer program)
Suggested Formative Assessment Practices/Processes
Teacher created quizzes Teacher created exit slips ***Height vs. Arm Span Activity*** **:** This activity asks students to gather bivariate data, create a scatter plot and line of best fit, identify the trend and relationship between the two sets of data, and draw conclusions with justification based on their findings.

Differentiation and Accommodations
• Provide graphic organizers • Provide additional examples and opportunities for repetition • Provide tutoring opportunities • Provide retesting opportunities after remediation (up to teacher and district discretion) • Teach for mastery not test • Teaching concepts in different modalities • Adjust homework assignments Note: Struggling students may need less data points to graph. They may need to use a piece of spaghetti to find the line of best fit. Higher level students may use their line and make predictions or make some general statements about the relationships.