Grade/Subject	Grade 7/ Mathematics Grade 7/Accelerated Mathematics
Unit Title	Unit 4: Proportional Reasoning
Overview of Unit	In this unit, students will apply and extend previous understandings of ratios, unit rates, proportional reasoning and percentages begun in the 6th grade. Students are now expected to be able to compute unit rates with non-whole numbers. Students previously only had to be able to formulate real world ratios given two quantities; now greater depth is required with students expressing proportional relationships within coordinate plane, recognizing the constant, formulating equations representing the situation and determining the graphical representation of singular unit rate. In addition, students need to use previous knowledge of setting up percentage proportions to find real world multistep solutions for taxes, interest, commissions, percentage increase/decrease, scale drawings, etc.
Pacing	Grade 7 Mathematics - 15 days Grade 7 Accelerated Mathematics - 27 days

Background Information For The Teacher
In sixth grade, students begin to formalize ratio as a particular kind of comparison between two quantities. A ratio is a comparison between two values of those quantities (numbers) that does not change if they are *varied multiplicatively by the same factor*. What is fundamentally different and new for students is the idea that the amount of two quantities can be in the same relationship (and hence be referred to as "equal") even though the amount of each of the two quantities gets larger or smaller. The understanding that equal means "preserving the particular kind of relationship" between the values is an important cognitive achievement for students in the seventh grade. A constant of proportionality in a particular context describes the ratio between two quantities is essential for students to keep track of the order of the proportional quantities. Students learn to be flexible, adaptable, precise, and sensitive to context in their problem solving. They express the constant of proportionality in either way as appropriate to the context. Geometric uses in using proportions are introduced in an additional real world situation in the use scales and creating maps. The most complex forms of problems with ratios and percents involve those with multiple steps. A fairly common one is when students must figure out "percent increase" or "percent decrease." Similar to the problematic with "more than" a percentage less than 100, English and mathematics spar

and it is important for students to consider the apparent contradictions in language and learn the different ways that, as a society, we conventionally describe percent increase and decrease.

Essential Questions (and Corresponding Big Ideas)	
Where do we use proportions and ratios in everyday life? • Whenever we compare two quantities there is a proportional relationship. How can I model unit rate and proportional relationships? • Using the coordinate plane we can determine the unit rate and the proportional relationship of two quantities. How can I use what I already know about percentages and proportions to find solutions to real world situations such as tax, commissions, percent increase/decrease or interest? • The method for finding the solution is the same as previous methods of setting up a proportion and finding the missing value; you first must identify the components of the proportion setting the part/whole = the percent/100. How can I use algebra to find the solution to proportional situations? • Total = the constant times the unknown quantity How can the use of proportions help in finding measurements in geometric situations? • Set up the proportion by comparing two given quantities and then similarly equating it to another ratio where there is an unknown amount.	
Core Content Standards	**Explanations and Examples**
7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. This standard focuses on computing unit rates using ratios of fractions	7.RP.1 If given the example of “a person walks 1/2 mile in each 1/4 hour”, students should be able to compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour and find that it is equivalently 2 miles per hour.

known as complex fractions. In a complex fraction, the numerator, denominator, or both are fractions. In the standard, $\frac{\frac{2}{2}}{\frac{1}{4}}$ is an example of a complex fraction. Complex fractions can be interpreted as division statements. For example, $\frac{\frac{2}{2}}{\frac{1}{4}}$ can be thought of as ½ divided by ¼.
Applications include situations where the quantities are measured in different units such as miles per hour, pounds per square foot, feet per second, and son on.

<u>What the **teacher** does:</u>

- Explore unit rates with ratios of fractions and compare them to unit rates with whole numbers from Grade 6.
- Treat complex fractions as division of fractions.
- Set up error analysis scenarios where students can identify errors in computing unit rates with complex fractions. For example, Homer calculated that if a person walks ½ mile every ¼ hour, the unit rate is 2 miles. However, Homer made an error. Find his error, correct it, and explain to Homer why 2 miles is not the correct answer.
- Provide opportunities for students to compute the unit rates in real-world problems.

<u>What the **students** do:</u>

- Discover that the structure of computing unit rates with whole numbers is the same concept as unit rates with ratios of fractions.
- Compute unit rates in real-world problems that involve complex fractions.
- In writing, explain the errors that can be made when computing unit rates with complex fractions and unlike units.

<u>Misconceptions and Common Errors:</u>

It is not uncommon to find seventh-grade students who are not fluent with fraction division. Sometimes the format of a complex fraction confuses them when they are used to seeing fraction division written horizontally as $\frac{1}{2} \div \frac{1}{4}$. Discuss how the division bar in the complex fraction means the same as the symbol ÷.

For students having difficulty understanding unit rate and those having trouble with different units such as miles per hour, pictures and diagrams may help. Use the example from this standard: If a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction $\frac{\frac{2}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour. Model with a diagram as shown. The bar represents 1 hour broken into ¼ hour segments.

¼ hour	¼ hour	¼ hour	¼ hour			
½ mile	½ mile	½ mile	½ mile			

For this diagram, students can see that the word problem is showing ½ mile every ¼ hours.

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

This standard emphasizes two methods for deciding whether a proportional relationship exits. One method is to use equivalent ratios in a table. If the ratios are equivalent, then you have a proportional relationship such as:

# of people in room	1	2	3	4	5
# of hands in room	2	4	6	8	?

The other method is to graph the relationship on a coordinate plane and observe whether the graph is a straight line that goes through the origin. Note that computation using cross-multiplication is not a part of this standard.

What the **teacher** does:

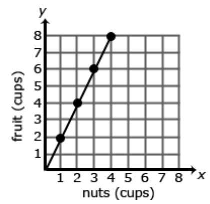
- Explore proportional reasoning scenarios with students to be sure they understand the meaning of proportional relationships in context before using the tables or graphs. While some number combinations may be proportional, the real-world example attached to the numbers my not be. Use examples and non-examples for students to identify and compare. An example is "2 music downloads cost $1.98; therefore, 4 music downloads cost $3.96." A non-example is: "Three boys can run a mile in 10 minutes; therefore, 6 boys can run a mile in 20 minutes."
- Ask students to write their own examples and non-examples of proportional relationships. Student's work can be shared and discussed.
- Discuss equivalent ratios with the students. Ask them to suggest some equivalent pairs. Relate to equivalent fractions. Display the pairs as the students suggest them in the form a/b = c/d, where b cannot equal zero and d cannot equal zero. Define two equivalent ratios as a proportion.

7.RP.2. Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the slope of the line.

Examples:

- A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

Serving Size	1	2	3	4
Cups of Nuts (x)	1	2	3	4
Cups of Fruit (y)	2	4	6	8



What the **students** do:

- Sort real-world examples of proportional relationships from non-examples. Students can create their own examples to demonstrate that they understand the concept of proportional relationships when there is a context attached. Discover that the
- Communicate orally and/or in writing that a proportion is a statement of two equivalent ratios. Students apply what they know about equivalent fractions to equivalent ratios.
- Model proportional relationships by creating tables: determine if a proportional relationship exists from a given table.
- Model relationships on graphs to determine if they are proportional.
- Test their hypotheses about whether a proportional relationship exists between any two points on the lines graphed. Students may draw the conclusion that all points on the line are proportional to all other points on the line by relying on tables, verbal statements, or logical arguments to draw the conclusion.

Misconceptions and Common Errors:

- Pose examples of proportions written with the quantities in different positions. Encourage students to decide if there is more than one correct way to setup a proportion. For example: "Set up a proportions showing that 3 out of 15 students are girls is the same ratio as 1 out of 5 students are girls." 3/15=1/5 or 15/3 = 5/1 or 1/3=5/15or 3/1=15/5 Ask students to explain how they know 15/1=5/3 is not a correct proportion for example. - Provide examples of equivalent and non-equivalent ratios to students for them to test with a table to decide if they are proportions. Conversely, present students with a table for a context and ask them to determine if all of the entries in the table are proportional. - Graph two ratios on a coordinate plane from a proportional scenario and look for a straight line that goes through the origin to determine if the two ratios are proportional. For example: "Maria sells necklaces and makes a profit of $6 for each necklace. How much money does she make for selling 3 necklaces?" - Pose the task to students: Select other points on the graphed line and determine if they are proportional. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. This standard focuses on proportional relationships that can be represented as tables, graphs, equations, diagrams, and verbal descriptions. Students have already seen tables, graphs, and verbal descriptions. The unit rate on a graph is the point where x=1. In an equation, it is the slope represented by the coefficient, m, in the formula y=mx+b. The terms unite rate, constant of proportionality, and slope are equivalent. Note that students are only required to read and interpret equations in this standard. <u>What the **teacher** does:</u> - Facilitate a discussion about representations of proportional relationships using a real-world scenario. For example, beginning with the verbal description: Mark was looking to fertilize his lawn, which is 432 sq. ft. He read the packages of 5 different fertilizer bags to see how much he should use. Bag A stated 2 ounces per 4 square feet, Bag B stated 4 ounces per 8 square feet, Bag C stated 1.5 ounces per 3	While graphing students may need to be reminded that the same types of quantities need to be graphed on the same axis. For example, when checking to determine if 10 cans of soda for $1 is proportional to 50 cans of soda for $10, the cans of soda must both be represented on the same axis. Ensure students are using graph paper or graphing calculators for all graphing. Remind them to label the axes. The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1). The constant of proportionality is shown in the first column of the

square feet, and Bag D stated 6 ounces per 12 square feet. Are these rates proportional? If yes, what is the unite rate? How much fertilizer does Mark need for his lawn?

- Using a real-world context have students determine if the relationship is proportional using graphs and/or tables. If it is proportional, facilitate a discussion with the class on the unit rate.
- Share a verbal description of proportional relationship and ask students to interpret it with a diagram such as bars. Encourage students to write how they interpreted the proportional relationship.
- Introduce equations as a statement of the proportional relationship. For fertilizer story the equation is f=2z, where f is the amount of fertilizer needed and z is the size of the lawn in square feet.
- Provide students with real-world proportional relationship expressed in a verbal description, graph, table, and equation. Challenge students to work with a partner t compare ow the unit rate is expressed in each representation. Share student discoveries in a large class discussion.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

In the previous standard students read equations to find the unit rates. In this standard students are given verbal descriptions of proportional relationships and are expected to create the equations in the form y=mx. For example, in Town C if you are caught speeding, you receive a traffic ticket. The penalty is $25 for every mile over the speed limit. What is the equation if p represents the penalty and m represents the number of miles over the speed limit? The equation is p =25m.

What the **teacher** does:

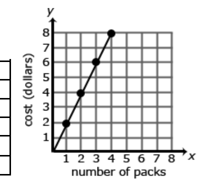
- Provide students with real-world proportional reasoning problems. Ask students to represent the stories as tables and graphs. Using the student-generated graphs and tables, create equations that model the proportional relationship.
- Provide students with real-world proportional relationship problems presented as tables, graphs, and verbal

table and by the slope of the line on the graph.

- The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.

Table:

Number of Packs of Gum (*g*)	Cost in Dollars (*d*)
0	0
1	2
2	4
3	6
4	8



Equation: $d = 2g$, where d is the cost in dollars and g is the packs of gum

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as "the number of packs of gum times the cost for each pack is the total cost in dollars". They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost.

What the **students** do:

- Model proportional relationships several different ways.
- Translate a proportional relationship from a verbal description into a diagram and explain in writing how the diagram shows a proportional relationship.
- Discover that the unit rate (constant of proportionality) is the numerical coefficient in the equation of a proportional relationship.

Misconceptions and Common Errors:

Finding the unit rate from a graph can be confusing. Some students cannot remember if the unit rate is the (1,y) or (x, 1) point. It is helpful to have a familiar unit rate students can recite such as 1 CD for $11.99 that helps them remember the x, which is first in a coordinate pair, is the 1 and the y is the unit rate.

descriptions. Have students write equations to model those relationships. • Provide opportunities for students to write about how they create equations that model proportional relationships. Some suggestions are exit slips, entrance slips. Letters and journals. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. An example of a proportional situation is: The scale on a map suggests that I centimeter represents an actual distance of 4 kilometers. The map distance between two towns is 8 centimeters. What is the actual distance? What the **teacher** does: • Present students with a verbal description of proportional relationship and build the graphical representation with the students. Be sure students give input on the labels for the x and y-axes. Facilitate a discussion about the graph with students asking the for the meaning of individual points and asking students to justify their responses. • Have students compare graphs that show proportional relationships and talk to a partner about what they notice. • Focus on points (0,0) and (1,r), the origin and the unit rate, respectively. • Use points that are not whole numbers and points where students need to estimate the coordinates.	What the **students** do: • Model proportional relationships presented as tables, verbal descriptions, and graphs in equation form. • Justify in writing the reasoning used in creating an equation for a given proportional relationship expressed verbally. Misconceptions and Common Errors: Some students confuse the variables in equations when they try to express the proportional relationship. It can be helpful to use letters closely representing what the variables stand for such as using f for fertilizer instead of x.

	What the **students** do: • Explain the meaning of a point on a graph in the context of the situation. Students should be able to explain examples with words such as, "Point (5,7) is the point that represents 5 health bars for $7.00" or "91, 10) represents the unit rate (constant of proportionality), meaning 1 teacher for every 10 students at the school board." • Discover the graphed proportional relationships are straight lines. Misconceptions and Common Errors: When finding points on a line that represents a real-world proportional relationship, students may think that the line stops at the origin. The teacher should show that the line continues into Quadrant III but that the points are not appropriate for the real world situation. For example, in a proportional relationship between the number of teachers and the number of students at grade level, it does not make sense to have -3 teachers.
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	7.RP.3 Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value. Examples: • Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs $4.17. What is the projected cost of a gallon of gas for April 2015?

In this standard students solve problems involving proportional relationships. Students set up and solve proportions using cross-multiplication. For example: "Directions to make a table cloth call for ¾ yard of ribbon for every 2 yards of fabric. If you increase the amount of fabric used to 3 yards, how much ribbon will be needed?" The proportion is $\frac{\frac{3}{4}}{\frac{2}{1}} = \frac{x}{3}$. To cross multiply: $3 * \frac{3}{4} = 2x$

Problems for this standard should be multi-step and include contexts with simple interest, tax, tips commissions, percent error, percent increase/decrease, discounts, fees, markups, markdowns, discount, sales, and/or original prices.

To calculate a percent increase from 2 to 10, find the difference between the two numbers, in this case, 10-2=8. Take the difference 8, and divided by the original number 8/2 = 4. Multiply the quotient by 100: 4 x 100 = 400%.

What the **teacher** does:

- Focus time on the vocabulary for this standard. Paper foldables, word walls, graphic organizers, using words in context, and writing stories all give students a chance to

  - A student might say: "The original cost of a gallon of gas is $4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of $4.17 is about $1.04, the projected cost of a gallon of gas should be around $9.40."

$4.17 + 4.17 + (0.24 • 4.17) = 2.24 x 4.17

100%	100%	24%
$4.17	$4.17	?

- A sweater is marked down 33%. Its original price was $37.50. What is the price of the sweater before sales tax?

$37.50 Original Price of Sweater	
33% of $37.50	67% of $37.50

  - The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.
- A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?

Discount 40% of original price	Sale Price - $12 60% of original price
Original Price (p)	

0.60p = 12

- At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution.
- A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his

clarify the meaning of these terms, which they may encounter in daily life but not fully understand. Bring in items familiar to students such as tennis shoes a six-pack of soda, and so on and use them to model situations that use the vocabulary. Vocabulary should include simple interest, tax, tip/gratuity, discount, commission, fees, sale, markup, markdown, and original price.

- Use cross-multiplication to solve problems involving proportional relationships. Use numbers in your problems that do not lend themselves easily to mental arithmetic.
- Begin with single-step problems and move to multi-step using a wide variety of contexts. Make use of everyday examples such as finding sales on line, in print media, and on TV.
- Ask students to write problems that can be solved with setting up proportions prompted by media ads.
- Introduce students to percent increase/decrease and percent error problems. Encourage students, through questioning, to discover the similarities among the formulas for these concepts.

goal?
After eating at a restaurant, your bill before tax is \$52.60 The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill. The amount paid = 0.20 x \$52.50 + 0.08 x \$52.50 = 0.28 x \$52.50

What the **students** do:

- Explore use of the vocabulary words in this standard by finding examples in the media and explain how they are used in each situation.
- Solve problems involving proportions using cross-multiplication.
- Solve problems involving percent error and percent increase/decrease.
- Use the structure of percent error and percent increase/decrease problems to explain how the formulas for these concepts are similar.

Misconceptions and Common Errors:

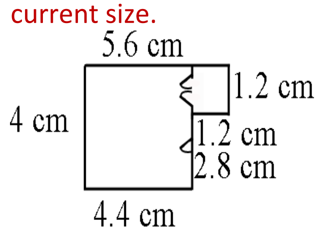
Students may have misconceptions about the vocabulary commonly used in the media such as sale, discount, and tax. It is important to discuss what students already know about these words in order to correct any pre-existing misconceptions. For individuals with difficulties with particular words, use graphic organizers such as the Frayer model. Acting out situations can help students remember certain steps. For example, acting out shopping for a pair of tennis shoes and a tennis racket and paying tax at the register will help students remember that tax is calculated on the cost of the total bill where the items bought need to be added up before tax is calculated.

7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Students work with scale drawings. They learn how to read them, calculate the scale, compute the actual lengths from the scale in the drawing, and reproduce a scale drawing using anther scale. Scale drawings are proportional to one another. Problems should center on experiences in the students' own lives. Examples include but are not limited to scale drawings of student rooms at home, the classroom, and comic book strips. The term scale factor should be used when students are asked to reproduce a scale drawing at a different scale. A sale factor is a number that multiplies some quantity. For example, doubling the length of a window that is 3 ft long corresponds to a scale factor of 2(2 X 3 = 6).

7.G.1. Example:

• Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size.



What the **teacher** does: - Pose a problem scenario that involves something familiar to students such as having to draw the classroom to scale for a set of blueprints. Allow students to do the measuring of the dimensions in the room and use this problem scenario to establish the basics about scale drawings and scale factors. - Assign projects similar to the classroom scenario to work on, such as scaling their bedrooms, the school cafeteria, and so on. Let students present them to the class. - Provide students with scale drawings and ask them to find the actual measures. Do one together as a class using a problem setting such as calculating the actual area of a space using a scale drawing. Pose questions such as the following: Does changing the scale factor affect the actual area? - Prepare problems where students need to change the scale of a drawing. Relate to proportional reasoning and ratios.	What the **students** do: - Read and create scale drawings from familiar settings. - Use precise mathematical language when presenting solutions to scale drawing problems to the class. - Calculate actual measures such as area, perimeter, and volume from scale drawings using appropriate measurement units. - Redraw a scale drawing using a different scale. Misconceptions and Common Errors: Have students use graph paper to make their scale drawings, as it will cut down on measurement errors. Students without a solid grasp of measurement units such as those for area will have difficulty with this standard, as will students who need more help with proportional reasoning. Take the opportunity while measuring the classroom or other hands-on measuring opportunity to reinforce measurement units for those students.
Standards for Mathematical Practice	**Explanations and Examples**
Analyze proportional relationships and use them to solve real-world and mathematical problems. **7.RP.1, 7.RP.2, 7.RP.3** These standards extend what students learn in Grade 6 about ratios to analyzing proportions and proportional relationships. Students calculate unit rates with complex fractions and move to recognizing and representing proportional relationships in equations and on graphs. These skills and understandings are used to solve multi-step ration and percent problems involving real-world scenarios such as interest, tax shopping sales, and so on. **MP1. Make sense of problems and persevere in solving them.** **MP3. Construct viable arguments and critique the reasoning of others.** **MP4. Model with mathematics.** **MP6. Attend to precision.** **Draw, construct, and describe geometrical figures and describe the**	Students solve multi-step and real-world percent problems. Students recognize proportional relationships from non-proportional ones and discuss their reasoning with others. Students learn to represent proportional relationships as tables, graphs, verbal descriptions, diagrams and equations. Students use units in their ratios requiring them to attend to the units such as 8 miles in 4 hours in a rate of 2 miles per hour.

relationships between them. **7.G.1** Students work with their hands drawing, constructing geometric shapes, and concentrating on triangles and building them given the three angle measures or the measure of side lengths. Students find relationships between figures such as the plane figures that result from slicing a three-dimensional figure. Using scale drawings, and students solve problems including finding the actual lengths from scale drawings or redrawing a scale drawing to another scale. **MP1. Make sense of problems and persevere in solving them.** **MP4. Model with mathematics.** **MP5. Use appropriate tools strategically.**	Students solve problems using scale drawings. Students use drawings and hands-on materials to model geometric shapes and relationships. Students draw free hand or use technology or other tools to draw geometric shapes.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*
• Dividing the denominator into the numerator results in finding the unit rate. • Graphing the coordinates of ratios will result in seeing if the ratios are proportional demonstrated by a straight line. • Proportional representations can be written as an equation by the total = the constant times the quantity. • In creating a graphic display of proportional relationships, when the x – coordinate is 1, the y – coordinate is my unit rate. • If I multiply the decimal equivalent of a percentage by the	• Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. • Decide whether two quantities are in a proportional relationship. • Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

whole, I can find the percent of a number. • If set up a proportion, I can find the percent by putting the part/whole = x/100. • Key's Actual Distance/Key's Scaling Distance = Actual Distance/Map Distance	• Represent proportional relationships by equations. • Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. • Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. • Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

UNDERSTAND

Big ideas, generalizations, principles, concepts, ideas that transfer across situations

- We can use proportions to find equivalent values.
- Ratios compare two items.
- Unit rate takes the ratio of two items being compared any gives the equivalent amount if there is only one unit being measured.
- If two comparisons are proportional then their ratios are equal.
- When we graph ratios we can recognize if it is proportional.
- I can make a ratio chart and plot the ratios.
- I can use proportions to solve interest, sales tax, tipping, and other percentages problems.
- I can use proportions to make a map correct in its dimensions.

Common Student Misconceptions for this Unit

Students may have the misconception that the unit rate for a relationship such as ¼ : ½ would be less than 1 due to both quantities being less than 1. If students estimate, they may round rational numbers to common benchmarks (1, ½, 0) and make wrong conclusions. For example, if two quantities are in a ratio of 7 ¾ : ½ ; students mistakenly round 7 ¾ to 8 and ½ to 1 and conclude that these quantities are in a ratio of 8:1.

Students often have the misconception that percents cannot exceed 100% or 100 : 100. It is important that they become comfortable with percents such as 200% and can simplify the ratio from 200 : 100 to 2 : 1. Likewise 150% implies a ratio of 1.5 : 1 and 325% implies a ratio of 3.25 :

1.

Another common misconception that emerges is that students believe that if an original value decreases by a certain percentage every year, the original value will eventually become 0 when the percentages add to 100. For example, if they are told that the value of a car $10,000 and it decreases by 25% per year, then it will be valueless in 4 years because 25% x 4 = 100%. Students should be challenged to carry out the operation in which they realize the 25% always acts on the value of the car at any given year so that:

After year 1: $10,000 * .25 = $2500
After year 2: $2500 * .25 = $625
After year 3: $625 * .25 = $156.25
After year 4: $156.25 * .25 = $39.06

Indeed, students should be posed the question: "If this continues, will the car ever be valueless?"

Unit Assessment/Performance Task	DOK
Unit 4 Test Unit 4 Performance Task "Speedy Texting"	

Vocabulary
• analyze • area • commission • constant rate of proportionality • coordinate plane • covariance – a measure of how related the variance are between two variables. The extent to which any two random variables change together or vary together.

- equivalent
- fraction
- gratuities or commissions
- mark downs
- mark ups
- origin
- percent error
- percent increase/decrease
- point
- proportion
- ratio
- simple interest
- unit rates

Key Learning Activities/Possible Lesson Focuses (order may vary)

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Lesson 1: Compute Unit Rates

Students will refine their skills from the 6th grade curriculum, now problem solving to find unit rates when given fraction/decimal quantities.

a) Students will demonstrate the solutions to finding unit rates when given two quantities including whole numbers, fractions, and decimals.

b) Students find unit rates within real world situations using whole and non-whole number quantities.

(Examples: finding the rate of speed distance/time, cost per unit). Students can be given scenarios where given different distances and times, they compare rates/speeds. Faced with two retail items students can determine the unit price and reason which is the better deal.

Potential Activity:
Shop Until You Drop- http://www.teachersfirst.com/winners/shopdrop.cfm

Lesson 2: Working with proportionality and unit rate and creating equation and graphical representations

Students work with knowledge that a ratio is comparing two items. Given tables or other data forms where two items are present, students can set up proportions to determine proportionality and these two items can therefore be graphed as the x and y coordinates. This results in proportional relationships being exposed as a linear graph. Students will scaffold their understanding of graphing ratios to determine proportionally to determine the unit rate. Students should be able to work with data interchangeably – taking the data and graphing, then determining the unit rate or having the unit rate first then graphing accordingly.

a) Using data, students working in groups setting up proportions from data. Students then can determine if lists are proportional prior to graphing, hypothesizing what they anticipate seeing before they graph. Then they verify proportionality by graphing.

Example:

Miles	45												
Hours	1	2	3	4	5								

Dollars	3.30												
Pounds	1	2	3	4	5								

b) Students will be given real world examples in table form. They will show work in determining whether the items are proportional. Then students explain the procedures they used in determining if proportional.

Finally, students graph to verify if they are proportional or not. (Examples: 4 cars traveling for various times and distances, are they traveling at same speed?)

c) Given graphs, tables, and data in written form students can identify the unit rate and write how they determined the unit rate and what patterns they are observing. Students should also be able to take the unit rate and devise a graph. This unit is the constant rate of change and the term constant should be stressed as it will be important for following concepts in this unit and further algebra topics.

Potential Lesson:
http://education.ti.com/calculators/timathnspired/US/Activities/Detail?sa=1008&t=9447&id=16897

d) After students have a working knowledge of what the constant rate of change is students can be introduced to writing equations being given written data, tables, or graphs. You can then incorporate writing equations into proportional table and graphing exercises. (Example: Given a recipe that calls for 3 ½ cups of flour for 2 servings, create a table of cups of flour needed for various servings for the students to prove whether the data is proportional, have them find their own list of cups needed for their own created number of servings. Then the students can identify given numerous graphs which apply to the situation or create their own. Finally the students can now create an equation for the situation.
e) Before lesson is complete students should be able to bring it all together by dissecting any written data, graph, equation, or table and be able to determine the constant/unit rate how the coordinates relate to ratios, how the ratios can be determined if proportional. Any point on a graph should be able to written as a ratio and find a proportional ratios to be graphed.

Sample Activity Below:

The monthly cost of Jazmine's cell phone plan is graphed on the grid below. Her friend Kiara selected a plan that charges $0.25 per text, with no monthly fee, because she only uses her phone for texting.

20
Cost 15
10
5
0

0 10 20 30 40 50 60 70 80 90 100

Number of Texts

a. Write an equation to represent the monthly cost of Kiara's plan for any number of texts.

b. Graph the monthly cost of Kiara's plan on the grid above.

c. Using the graphs above, explain the meaning of the following coordinate pairs:

i. (0, 20): ____

ii. (0, 0): ____

iii. (10, 2.5): ____

iv. (100, 25): ____

d. When one of the girls doubles the number of texts she sends, the cost doubles as well. Who is it?

Explain in writing how you know.

Lesson 3: Using Proportions to Solve Real Life Percentage Situations

Students at this point should be able to use the 6th grade proportional curriculum understanding and this unit's lessons thus far to set up a proper proportion with percentages. Students will need to find the percent given a situation. In particular, real world situations of simple interest, taxes, sales pricing, tipping, and percent increase, or error are of importance.

a) Students compare measurements taken to true measurements in order to work with percent error. First students find the difference between the actual and recorded data and then find the percentage error.

b) Given real world situations students can determine the amount of interest, taxes, and discount or mark up in a given situation using proportions. Students should also be able to find the percentage given the original amount and the interest, tax, or discount or mark up. (For example finding the amount to leave on a restaurant bill for an 18% gratuity, taking a tip amount and determining the percentage of tip left on a given total, or taking the percentage gratuity and tip amount and determining the original bill).

Potential Lessons:
http://www.mathgoodies.com/lessons/percent/sale_price.html
http://www.mathgoodies.com/lessons/percent/interest.html
http://www.mathgoodies.com/lessons/percent/commission.html
http://www.mathgoodies.com/lessons/percent/sales_tax.html
http://www.mathgoodies.com/lessons/percent/change.html

Lesson 4: Using Proportions to Create Maps and Scale Drawings (6 Days)

Students use their understanding in setting up proportions to scale and create maps and drawings that are proportional. Students use the key or given information to set up a proportion comparing the scaling factors to the real life measurements.

a) Students should first assure they can properly set up the proportions with scaling activities.
b) Once students can set up proportions properly they can begin to attempt to work in reverse given values on a map, determining the scaling and finding the actual or map measurements and then attempt to create map or objects given a proportional relationship.

Potential lessons:
http://fcit.usf.edu/math/lessons/activities/basebalT.htm
http://fcit.usf.edu/math/lessons/activities/ShadowT.htm
http://www.asset.asu.edu/new/mathactive/lessons/166/lesson166.swf

Supplemental Materials and Resources

Online Lessons & Tasks

- Increasing and Decreasing Quantities by a Percent
  http://map.mathshell.org/materials/lessons.php?taskid=210&subpage=concept
- Ice Cream
  http://map.mathshell.org/materials/tasks.php?taskid=389&subpage=expert
- 25% Sale

http://map.mathshell.org/materials/tasks.php?taskid=358&subpage=apprentice

- Buses
  http://map.mathshell.org/materials/tasks.php?taskid=365&subpage=apprentice
- Short Tasks - Ratios and Proportional Relationships
  http://map.mathshell.org/materials/tasks.php?taskid=397&subpage=novice
- Highway Robbery
  http://illuminations.nctm.org/LessonDetail.aspx?id=L838
- Now and Then
  http://illuminations.nctm.org/LessonDetail.aspx?id=L837
- What's Your Rate?
  http://illuminations.nctm.org/LessonDetail.aspx?ID=L511
- Ratio and Percents
  http://www.lausd.net/math/InstructionalGuides/Lessons/RATIO_AND_PERCENTS.pdf
- Problems with Percents
  http://www.lausd.net/math/InstructionalGuides/Lessons/PROBLEMS_WITH_PERCENTS.pdf
- Gauging Gas Mileage
  http://www.lausd.net/math/InstructionalGuides/Lessons/GAUGING_GAS_MILEAGE.pdf
- Calling Plans
  http://www.lausd.net/math/InstructionalGuides/Subjects/G7/2009%20MIG/CallingPlan%20Grade_7_LessonPlan.pdf

Worksheets

- Proportion Worksheet
  http://www.mathworksheets4kids.com/ratio/proportional.pdf
- Are the Ratios Proportional?
  http://www.teach-nology.com/worksheets/math/ratios/ratioandprop19.pdf
- Shopping with Proportions
  http://www.teach-nology.com/worksheets/math/ratios/ratioandprop24.pdf
- Unit Rates – Comparisons
  http://chatt.hdsb.ca/online%20conferences/mfm1p/Task1/pdfs/worksheet1-3.pdf
- Ratios and Unit Rates Worksheet
  http://www.shmoop.com/pre-algebra/handouts/ratios-percentages/ratios-and-unit-rates-questions.pdf

- Scale Drawings
  http://www.ohs.osceola.k12.fl.us/staff/websites/pettettl/documents/GeometrySCALEDRAWINGSpp.174-180.pdf

Videos

- Unit Rates (Pearson)
  http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-890s.html
- Proportion and Direct Variation
  http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-196s.html

SMART Board Lessons

Online Interactive Activities & Games

- Ratios & Proportions Worksheets (online interactive)
  http://www.mathslice.com/actionctl.php?actionid=542
- Unit Rate Worksheet (online interactive)
  http://worksheets.tutorvista.com/unit-rate-worksheet.html
- Scale Factor
  - http://illuminations.nctm.org/ActivityDetail.aspx?ID=176

Literature connection:

"Many books and stories discuss comparative sizes; concepts of scale as in maps; giants and miniature people who are proportional to regular people; comparative rates, especially rate, especially rates of speed; and so on. A book may not appear to explore proportions, and the author many not have had that in mind at all, but comparisons are the stuff of many excellent stories and are at the heart of proportional ideas". (Van De Walle, J. (2004). *Elementary and Middle School Mathematics, Teaching Developmentally*. Fifth Edition. Boston: Pearson Education, Inc.)

Below are some suggestions:

If You Hopped Like a Frog, by David Schwartz, 1999

Counting on Frank, by Rod Clement, 1991

The Borrowers, Mary Norton, 1953

Proportional Relationships - The Principal's New Clothes - Topics: permutations; ratio; proportion http://sci.tamucc.edu/~eyoung/Literature%20webpages/Leonor/principal_new_clothes.html

Interdisciplinary connections:

Science

- Scientists use ratio tables to find the amount of equivalent amounts are needed for greater or lesson quantities. Example if two Hydrogen atoms are present in 1 molecule of water then there are 14 water atoms in 7 molecules of water.
- The golden ratio and its appearance in nature. You can find the golden ratio in nature in some flowers, fruits, and vegetables.
- Allometry is the study of how these processes scale with body size and with each other, and the impact this has on ecology and evolution.

Social Studies/Geography

- Students can create maps of school, hometown, etc, using scaling techniques.
- Students can find percentages of demographic using data and vice versa.

Tools/Manipulatives

- Graph Paper
- Rulers
- Calculator
- Common objects such as tennis shoes, cereal boxes, etc.
- Copies of restaurant menus

Suggested Formative Assessment Practices/Processes

- Teacher created quizzes
- Learning Log – where students access their understanding 1-5 (5 highest, 1 lowest) and then give a sentence or two about what did or did not understand.

Use of three color cups (Green, Yellow, Red) for students to describe their level of understanding: Green cup = no problem, I understand what to do; Yellow cup= I need some assistance but it is not stopping me; Red cup= I need help

Differentiation and Accommodations

- Provide graphic organizers
- Provide additional examples and opportunities for repetition
- Provide tutoring opportunities
- Provide retesting opportunities after remediation (up to teacher and district discretion)
- Teach for mastery not test
- Teaching concepts in different modalities
- Adjust homework assignments