Grade/Subject	Grade 7/ Mathematics Grade 7/Accelerated Mathematics
Unit Title	Unit 2: Operating with Rational Numbers (Multiplication/Division)
Overview of Unit	In this unit, students will apply and extend previous understandings of operations with fractions to multiply, and divide positive and negative rational numbers, including integers, use properties of operations to generate equivalent expressions, and solve real-life and mathematical problems using both positive and negative rational numbers.
Pacing	Grade 7 Mathematics - 15 days Grade 7 Accelerated Mathematics – Units 1 and 2 may take 46 days.

Background Information For The Teacher
Students have studied operations with whole numbers, fractions, and decimals in previous grades. In this unit, students should extend this understanding to operations with all rational numbers including negative numbers. Exploring ideas about negative numbers by building and connecting to what students already know will not only help develop understandings of negative numbers, but also deepen understanding of meaning and operations of positive numbers, doing this will require students making meaning of the operations and analyzing what kinds of situations call for which operation. Students come to this unit having already informally experienced positive and negative numbers in their everyday lives—a repeated gain or loss of yardage in a football game, TV game shows in which participants lose points if they answer incorrectly, and sports teams being ahead or behind by some amount. This unit recommends exploring situations that require students to reason and represent with rational numbers including integers. Students have already explored the use of a number line as a wonderful model for developing understanding of order, for comparing rational numbers, as well as for developing the concept of opposites, distances, and absolute value. They have modeled addition and subtraction on the number line in the previous unit. The number line can also be used to model multiplication of integers. The inverse relationship between multiplication and division need to be addressed to help students generalize algorithms for the operations as well as looking at number patterns. Asking questions about meaning and about what makes sense will help focus students' attention on the situation, the operation and connections.

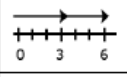
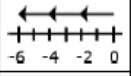
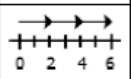
Essential Questions (and Corresponding Big Ideas)
When am I going to use positive and negative numbers? • Positive and negative rational numbers help us understand more real world situations. How can I model multiplying and dividing negative numbers? • A variety of math tools and real world examples can model negatives, such as number lines, two-color chips, thermometers, distance, altitude, underground parking and scenarios such as credits and debits. How can I use what I already know about multiplying and dividing positive numbers to multiply and divide with negative numbers? • The methods for multiplying and dividing whole numbers can be applied to multiplying and dividing other rational numbers. What rules can we find to generalize patterns when multiplying and dividing positive and negative numbers? • Real world examples can lead to a generalization of the patterns for multiplying and dividing integers. How can the use of mental math and estimation strategies help determine the reasonableness of answers? • Fluency in mental math and estimating enhances the ability to solve real world situations requiring the application of positive and negative rational numbers.

Core Content Standards	Explanations and Examples
7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational	7.NS.2**.** Multiplication and division of integers is an extension of multiplication and division of whole numbers. Examples: • Examine the family of equations. What patterns do you see? Create a model and context for each of the products. Write and model the family of equations related to 3 x 4 = 12.

numbers by describing real- world contexts.

Real world contexts help students make sense of multiplication of rational numbers. For example, it makes sense that 4 x (-6.50) = -26 when the context for this equation is Janene owes $6.50 to each of 4 people. How much does Janene owe altogether?

It is common to read and understand (-6) as "the opposite of six" as well as "negative six." Use "the opposite of" wording to make sense of equations such as (-2) x (-5) so that we read "the opposite of 2 times negative 5" or ((-1) x 2) x (-5) = -(2x-5) = -(-10) = 10.
Students should discover the rules for multiplying signed numbers, and the rules make more sense when given context. For example, the chart on the side shows equations with context.

Equation	Number Line Model	Context
2 x 3 = 6		Selling two packages of apples at $3.00 per pack
2 x -3 = -6		Spending 3 dollars each on 2 packages of apples
-2 x 3 = -6		Owing 2 dollars to each of your three friends
-2 x -3 = 6		Forgiving 3 debts of $2.00 each

What the Teacher does:

- Give several examples of multiplication with rational numbers to students in real-world contexts where the signs of the answers make sense. Examples can include, "I owe $6.50 to each of 4 people. How much money do I owe altogether?" 4 x (-6.50) = (-26). After several stories for each combination of signs (++. + -. --), ask students to suggest rules for multiplying rational numbers.
- It is common to read and understand (-6) as "the opposite of six" as well as "negative six." Use "the opposite of" wording to make sense of equations such as (-2) x (-5) so that we read "the opposite of 2 times negative 5."
- Help students generalize rules for multiplying signed numbers from tables of related facts.
- Provide students with an opportunity to determine which properties of multiplication hold true for rational numbers. This can be accomplished by giving students a list of properties and asking them to write multiplication equations with rational numbers that make use of the properties or giving students examples of multiplication equations with rational numbers and asking students which properties are shown the equations.
- Provide opportunities for students to write the rules for multiplication of signed numbers or answer questions about properties in journals, letters, and exit and/or entrance slips.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a

What the Students do:

- Discover the rules for multiplying rational numbers by reasoning from real-world examples.
- Use different interpretations for the (-) sign such as "negative" or "the opposite of" to make sense of real-world contexts using rational numbers.
- Conclude that the properties of the operations for multiplication hold for rational number multiplications.

Misconceptions and Common Errors:
Students who have difficulty generalizing rules for multiplication from real-world contexts can be given a mathematical context. Provide a number of equations with like signs and have students find the answers with a calculator. From there students can formulate a rule for multiplying rational numbers with like signs from a pattern they can see in the equations they solve. Repeat with equations having unlike signs.

rational number. If *p* and *q* are integers, then –(*p*/*q*) = (–*p*)/*q* = *p*/(–*q*). Interpret quotients of rational numbers by describing real-world contexts.

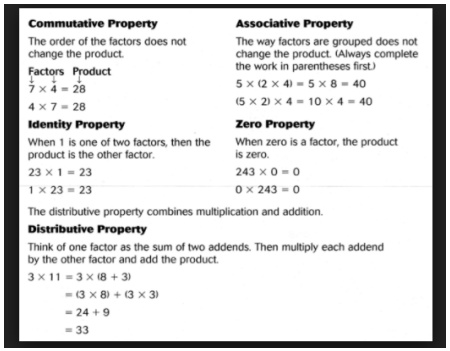
Division of rational numbers can be thought of as the inverse of multiplication relying on previous understandings of the relationship between multiplication and division. For example, (-25) divided by 5 = -5 because 5 x (-5) = -25. This preserves the relationship between multiplication and division found with whole numbers, including the fact that division by 0 is undefined. One explanation is $x \cdot 0 = 5, so\ 5 \div 0 = x$. There is not possible number for x. The equation $-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}$ is for the teacher, not the students. Use both $p \div (-q) and\ \frac{p}{-q}$ notations for division.

What the Teacher does:

- Relate the meaning of division with signed numbers to the meaning of division with whole numbers by using the inverse relationship of multiplication and division such as, " Since 7 x 3 = 21, we know that $21 \div 3 = 7$. Following the same reasoning, 7 x (-3) = (-21) so (-21)$\div (-3) = 7$."
- Expose students to many examples of division with integers so they can discover/establish formal rules.
- Use multiplication and division as inverse operations. As students to work in groups to explain why it is not possible to divide by zero. Encourage them to divide by zero and see what happens. One explanation is $x \cdot 0 = t, so\ 5 \div 0 = x$. There is no possible number for x.
- Use both p$\div (-q) and\ \frac{p}{-q}$ notations for division.
- Challenge students with questions such as, "-5 is the quotient. What is the equation?"
- Provide real-world contexts where quotients of rational numbers can be interpreted for example, if the temperature on Sunday was 0°F and 7 days later it was -21°F, what does -3 represent? How was -3 calculated?
- Provide opportunities for students to create real-world problems using division of rational numbers.
- Use writing as a formative assessment to learn about students' understanding of multiplication and division of rational numbers.
- Provide on-going practice with multiplication and division of rational numbers in real-world and mathematical contexts so that students may develop fluency.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

Present problems in real-world contexts that allow students to see the meaning of the properties of the operations. Properties include:



Commutative Property

The order of the factors does not change the product.

Factors Product

$7 \times 4 = 28$

$4 \times 7 = 28$

Associative Property

The way factors are grouped does not change the product. (Always complete the work in parentheses first.)

$5 \times (2 \times 4) = 5 \times 8 = 40$

$(5 \times 2) \times 4 = 10 \times 4 = 40$

Identity Property

When 1 is one of two factors, then the product is the other factor.

$23 \times 1 = 23$

$1 \times 23 = 23$

Zero Property

When zero is a factor, the product is zero.

$243 \times 0 = 0$

$0 \times 243 = 0$

The distributive property combines multiplication and addition.

Distributive Property

Think of one factor as the sum of two addends. Then multiply each addend by the other factor and add the product.

$3 \times 11 = 3 \times (8 + 3)$

$= (3 \times 8) + (3 \times 3)$

$= 24 + 9$

$= 33$

What the Teacher does:

- Use real-world contexts as examples for students to reason using properties of the operations. Let students solve in pairs and explain the operations they used such as:
  - Maria needed to borrow 1 cup of sugar to finish the icing on her cake. Her mother had 2 containers with a ½ cup of sugar in each. Her father had ½ of a 2 cup bag. Explain why it does not matter whose sugar Maria borrows.
  - Ms. Core bought stock in Harley Davidson Motorcycles. For the first 3 days she owned the stock, the shares changed rice by -2 ¼ a day. For the next 5 days, it changed +3/4 per day. Did Ms. Core show a profit during the first 8 days that she owned the stock? Would there be a difference if the stock changed +3/4 for the first 5 days and changed -1 2/4 the next 3 days? Explain your reasoning mathematically.
  - Encourage the use of precise mathematical vocabulary as students explain their reasoning.
  -

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

What the Students do:

- Use reasoning to determine that division by zero is not defined.
- Discover that division as the inverse of multiplication holds true with integers.
- Generalize rules for division with signed numbers from examples.
- Use p$\div (-q)$ and $\frac{p}{-q}$ notations interchangeably.
- Interpret a rational quotient in a real-world context.
- Clarify their own understanding of the relationship between multiplication and division of rational numbers through writing.
- Develop fluency through practice with multiplication and division of rational numbers.

Misconceptions and Common Errors:

Students who are not fluent with basic multiplication and division facts will have difficulty performing fluently with rational numbers. These students need additional practice. Many computer-based programs have success with basic fact mastery.

For students who do not understand why division by 0 is undefined, give specific examples of the relationship between multiplication and division that would not make sense. For example, $x * 0 = 5, so\ 5 \div 0 = x.\ There\ is\ no\ possible\ number\ for\ x.$

To convert rational numbers in fraction form to decimal form, use the meaning of fractions as division. For example, 4/5 = 4 ÷ 5. Using long division, $4 \div 5 = 0.80$. From repeated examples, students learn that the decimal form either ends in 0s (as in the example) or repeats digits/sets of digits. Students learn to use the bar above a digit/set of digits to designate digits that repeat. For example, 2/3 = 2 ÷ 3 = $0.\overline{6}$ and $\frac{39}{99} = 0.\overline{39}$.
This prepares students to learn about irrational numbers in Grade 8.

What the Teacher does:

- Define a/b as a divided by b. This may be a new understanding of fraction notation for many students.
- Use rational numbers that will end in 0s when converted to decimals such as ¾ = 0.75 and 33/2 = 22.50.
- Provide students with fraction sets to convert such as the thirds, sevenths, ninths, and so on. Using long division, students will find patterns of repeating digits. Use the bar over the first digit or set of digits that repeat such as 1/7 = $0.\overline{142857}$.
- Provide students opportunities to reflect on why they think the patterns are occurring in the thirds, sevenths, and so on. Opportunities may be in writing or conversations with classmates.
- Pose the following question to the class. Can you group the decimals forms of rational numbers into types? What do you notice?

What the Students do:

- Use properties of the operations to explain the solutions to real-world problems.
- Clarify their understanding of properties of operations by discussing with their partners and explaining their reasoning to the class.
- Practice use of appropriate mathematical vocabulary in discussions. Vocabulary included the names of the properties.

Misconceptions and Common Errors:
Students often confuse the associate property with the distributive because both properties use parentheses. This is a common error for students who do not understand what is happening in the equation. Try modeling the properties with simple examples such as this one for the distributive property I have 3 tetras and 4 goldfish in each of my 5 fish tanks. How many fish do I have?

5(3 + 4) = (5 X 3) + (5 X 4)

	What the Students do: • Use long division to convert rational numbers in fraction form to decimal form. • Explain why and how they know a long division quotient will repeat such as in the case of 1/7. • Sort the decimals form of rational numbers into two types, ending in 0/terminating or repeating. Students conclude that rational numbers can be written as repeating or terminating decimals. Misconceptions and Common Errors: Some students may have difficulty with long division. Look for patterns in repeated errors made by these students and target these specific errors. Common division errors are forgetting to add zeros in the dividend and placing the decimal point in the wrong place in the quotient Try some alternate algorithms that focus on place value. Some students may have tracking difficulties. Doing long division on graph paper is a solution for many students. Perseverance is important in converting to decimal form if there are many digits that repeat. Students should be encouraged to continue until they find a repeating patter or 0 because it is a common error that they stop the division algorithm too soon.

	Calculators are a useful tool for decimals that have over four digits that repeat. Sometimes students want to use the repeat bar over two or three repeating digits when only one digit repeats such as 3/9 = $0.\overline{3}$, not $0.\overline{333}$. This error can be corrected through the use of error analysis exercises where students are presented with incorrect work done by a fictitious student. The problems should re-create a common error or misconception and the students try to identify it, thereby clarifying their own thinking In this case, an example is as follows: Mario wrote the answer to a problem as 1/3 = $0.\overline{333}$. He made an error. Correct his mistake and explain to Mario how to avoid making the same mistake in the future.
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) Extend the work with order of operations to all rational numbers. An example of a mathematical problem is $-3\ x\ 2(\frac{5}{6} + -\frac{1}{2})$ = -2. Complex fractions are fractions with a fraction in the numerator and/or a fraction in the denominator such as $\frac{\frac{3}{4}}{\frac{1}{2}}$. Interpret the division bar to turn a complex fraction into division: $\frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{4} \div \frac{1}{2}$. <u>What the **teacher** does:</u> • Extend the work with order of operations to all rational numbers. • Provide real-world problems that build on previously studied skills. For example: "You want to buy a new tablet. The service agreement will deduct $22.50 from your savings every month to pay for it. How much will the deductions be at the end of the year?" 12(-22.50) = -270. Include problems that apply all four operations with rational numbers and complex fractions.	7.NS.3 Examples: • Your cell phone bill is automatically deducting $32 from your bank account every month. How much will the deductions total for the year? -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12 (-32) • It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent? $\frac{-100 \text{ feet}}{20 \text{ seconds}} = \frac{-5 \text{ feet}}{1 \text{ second}} = -5 \text{ ft/sec}$ <u>What the **students** do:</u> • Apply operations with rational numbers to problems that involve the order of operations. • Solve mathematical problems that use the four operations with rational numbers. • Solve real-world problems that involve the four operations with rational numbers. • Compute with complex fractions. <u>Misconceptions and Common Errors:</u> As equations become longer with more terms and more complex using rational numbers, some students are overwhelmed and do not know where to begin. Help these students by reviewing the order of operations and demonstrating how to solve equations one step at a time. Flip books created by students that do a step-by-step breakdown of a computation aid some students. For such a book, students can begin with a problem and perform one step on the first page, then repeat that step and add a second step to next page, continuing in this manner.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5 percent" is the same as "multiply by 1.05."*

Using equivalent expressions from the previous standard, focuses on how writing an equivalent statement can better show the relationship among the terms in the expressions. For example, 6x + 15 = 3(2x+5) means that three groups of 2x + 5 is the same as one group of 6x and 15.

What the Teacher Does:

- Present students with real world problems that can be modeled with more than one expression. For example: An item that is on sale for 20% off costs 80% of the original price. Write an expression by using x as the original price. Allow students to explain their expressions, decide if one another's expressions are equivalent, and explain how a particular expression relates the quantities in the problem. This can be done individually, in groups or as projects.

7.EE.2 Examples:

- Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents the weekly wages of both if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week? Can you write the expression in another way?

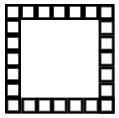
Students may create several different expressions depending upon how they group the quantities in the problem.

One student might say: To find the total wage, I would first multiply the number of hours Jamie worked by 9. Then I would multiply the number of hours Ted worked by 9. I would add these two values with the $27 overtime to find the total wages for the week. The student would write the expression. $9J + 9T + 27$.

Another student might say: To find the total wages, I would add the number of hours that Ted and Jamie worked. I would multiply the total number of hours worked by 9. I would then add the overtime to that value to get the total wages for the week. The student would write the expression $9(J + T) + 27$.

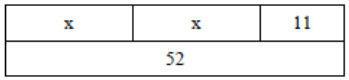
A third student might say: To find the total wages, I would need to figure out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, I would multiply the number of hours she worked by 9. To figure out Ted's wages, I would multiply the number of hours he worked by 9 and then add the $27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages. The student would write the expression $(9J) + (9T + 27)$.

- Given a square pool as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression do you think is most useful? Explain your thinking.



What the Students Do:

- Model contextual problems with multiple variable expressions.
- Explain orally and/or in writing, using precise mathematical vocabulary, how two equivalent expressions relate the quantities.

	Misconceptions and Common Errors: Many students have difficulty seeing that expressions are equivalent when the expressions are out of context. Use simple contexts so that students can reason with a context to explain why two expressions are equivalent. For example: Write two equivalents expressions for the following situation: All music downloads are 99 cents today. Maria wants to download 2 R&B hits, 1 rap hit and 3 hits by her favorite artist. Two equivalent expressions are 6 x .99 and (2 x .99) + (1 x .99) + (3 x .99). Focus student attention on how 6 hits for .99 cents each is the same as 2 hits and 1 hit and 3 hits for .99 each.
7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10 percent raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* Students solve multi-step real-world and mathematical problems. The problems should contain a combination of whole numbers, positive and negative integers, fractions, and decimals. Students will apply what they learned in previous standards about converting fractions, decimals, and percents and use properties of operations to find equivalent forms of expressions as needed. Students will be expected to check their work for reasonableness using estimation strategies. What the Teacher Does: • Pose a variety of multistep real-world and mathematical problems to solve, including integers, fractions, decimals, and percents. Students should convert fractions, decimals, and percents as in the example in the Standard where a	7.EE.3 Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations. Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate). Example: • The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.  $2x + 11 = 52$ $2x = 41$ $x = \$20.5$ What the Students Do: • Solve multi-step real-world and mathematical problems with precision. • Select an appropriate estimation strategy and apply it to a problem. Values in problems lend themselves to different

10% raise was interpreted as $\frac{1}{10}$ of the base salary. • Encourage the use of rounding, compatible numbers, and benchmark numbers to check for reasonableness of results. • Expect students to use a check for reasonableness on every problem. Have them explain orally and/or in writing their estimation strategies for some of the problems using journals or on exit slips.	strategies. • Justify the estimation process used by explaining, orally and/or in writing, how it proved their answer to be reasonable. If the estimate did not show an answer to be reasonable, explain how it helped lead to an accurate answer. <u>Misconceptions and Common Errors:</u> It is common for students to have difficulty with multi-step problems. Scaffold the problems by adding a question mid-way. Display the first step of the problem, allow students to find the answer, and then present the next part that relies on the first step. Gradually remove the middle question as students get used to finding a middle question and identifying it themselves. For example: Fred goes out to eat and buys a pizza that costs $12.75, including .50 tax. He wants to leave a tip based on the cost of the food. What must Fred do? First present the following: Fred goes out to eat and buys a pizza that costs $12.75, including .50 tax. How much did the pizza cost? Solve this part of the problem. Then, using the answer from Part 1, introduce the second part of the problem. He wants to leave a tip based on the cost of the food. What must Fred do? Some students' work may indicate a weakness representing numbers in different forms such as 10% as $\frac{1}{10}$. These students need additional practice. Use number lines, visuals such as bars, and hands-on materials instead of memorizing rules.
Standards for Mathematical Practice	**Explanations and Examples**
Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divided rational numbers. 7.NS.1, 7.NS.3 This cluster is about understanding and computing with rational numbers. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. Students learn how to add, subtract, multiply, and divided integers and apply properties of operations as strategies for each operation. Students journey from exploring the operations to formalizing rules. Students convert rational numbers to decimal form using division. The understanding of a rational number as one that terminates or repeats is covered in Grade 7 as preparation for the introduction of irrational numbers in Grade 8. **MP4. Model with mathematics.** **MP6. Attend to precision.** **MP7. Look for and make use of structure.** **MP8. Look for and express regularity in repeated reasoning.** **Use properties of operations to generate equivalent expressions. 7.EE2** Students apply properties of operations previously learned as	Students use multiple strategies to demonstrate the same meaning of an operation which include modeling with manipulatives or a on a number line. Students are working towards being independent thinkers by self-correcting any errors they may find. Students make use of what they already know about operations and their properties and extend the understanding rational numbers. Students use examples of integer multiplication to generalize a general rule.

strategies to add, subtract factor and expand linear equations that have rational coefficients. This skill leads to students being able to rewrite expressions in different forms so they can solve contextual problems and understand how the quantities in the problem are related. **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.** **MP6. Attend to precision.**	Students write expressions in different forms to understand how quantities in an equation are related. Students write expressions and equations to model contextual problems. Students communicate their reasoning using precise mathematical vocabulary.
Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **7.EE.3** Students focus on solving real-world problems and learn to use equations and inequalities to solve the problems by reasoning about quantities. Students learn to solve equations in the form $px + q = r$ and $p(x + q) = r$ fluently through practice. They compare algebraic solutions to arithmetic ones to demonstrate that they understand the sequence of operations in each approach and how they are the same and different. For inequalities, students graph solutions and then describe the solutions in terms of the context of the problem. **MP1. Make sense of problems and persevere in solving them.** **MP2. Reason abstractly.** **MP4. Model with mathematics.** **MP6. Attend to precision.**	Students solve multi-step real-world mathematical problems. Students use equations and inequalities to solve problems. Students solve problems by reasoning about quantities. Students write equations to model contextual problems. Students estimate answers to problems as a check to accurate solutions

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*
• Rules for multiplication and division of positive and negative numbers • Properties of operations (Distributive Property, Commutative, Associative, and Identity Properties of Addition and Multiplication, Additive Inverse Property) • Strategies for converting a rational number into a decimal • The decimal form of a rational number terminates in zeros or eventually repeats • Opposites and absolute value of rational numbers • A negative number can also be interpreted as the opposite of the positive number. (Ex: -5 can be interpreted as the opposite of 5.) • Computation with integers is an extension of computation with fractions and decimals • Models: Number line, chip model, area model, arrays, bar model, fraction circles, picture/visual • Estimation as a means for predicting & assessing the reasonableness of a solution. • Mental math and estimation strategies (benchmarks, clustering, rounding, front end estimation, compatible numbers) • Rational numbers are a set of numbers that includes the whole numbers, integers, as well as numbers that can be written as quotient of two integers, a ÷ b, where b≠0. • Rational numbers can be represented as fractions, decimals, & percents in infinitely many equivalent forms. • Expressions can be rewritten in different forms	• Multiply and Divide rational numbers • Develop and apply rules for multiplying & dividing signed numbers • Apply properties of operations as strategies • Solve multi-step problems in context • Convert between equivalent forms of rational numbers • Understand the relationship between equivalent forms of an expression • Use mental computation and estimation strategies • Assess reasonableness of answers.

• Rational numbers have multiple interpretations, and making sense of them depends on identifying the unit. • The concept of *unit* is fundamental to the interpretation of rational numbers.	

UNDERSTAND

Big ideas, generalizations, principles, concepts, ideas that transfer across situations

Rational numbers are a subset of the number system including & beyond whole numbers.

Properties of whole number operations can be applied to solving real world and mathematical problems involving rational numbers, including integers.

Integers can be divided, provided that the divisor is not zero, and every quotient of integers is a rational number.

Estimation and mental math are more complex with rational numbers.

Common Student Misconceptions for this Unit

- Having been taught how to add and subtract integers/rational numbers students apply those rules to multiplication and division.
- Students in earlier grades have been taught that when you multiply your answer becomes larger and when you divide it becomes smaller, which is not necessarily the case when working with fractions or decimals (examples: 8 x ¼ = 2, 30 ÷ 0.75 = 40). Also, when multiplying by a negative and positive number the greater the positive number results in a smaller value. (Example 7 x -5 = -35, 8 x -5 = -40, the latter is greater because its greater value to have only 7 groups of negatives as opposed to 8 groups).
- When using the distributive property students often don't recognize – as a negative and get confused when multiplying by a negative number. (Example of common misconception: -3 (x – 4) = -3x -12.
- With long division decimal conversion, students may reverse the placement of the dividend and divisor (example: ¾ = 4 ÷ 3 = 1.33333….).
- When estimating students only know how to round to whole numbers hence, since fractions less than ½ are closer to zero they estimate them to 0 in their calculations.

Unit Assessment/Performance Task	DOK

Unit 2 Test (Includes Unit 1) Unit 2 Performance Task "Cat Food"	

Vocabulary
• Absolute Value • Equation • Expression • Integer • Irrational Numbers • Multiplicative Inverse/Reciprocal • Order of Operations • Properties of Operations (Commutative Property, Distributive Property, Additive Inverse, Multiplicative Identity, Multiplicative Property of Zero) • Rational Number **Other terms students should know:** • Deposit • Expense, Loss • Income, Profit • Product • Quotient • Withdrawal
Key Learning Activities/Possible Lesson Focuses (order may vary)
These are ideas for lessons. **Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)** **Lesson 1: Simple operations with Integers** *Working cooperatively, students will develop the concepts of integer operations through investigation and*

practice using the number line and manipulatives.

a) Students will demonstrate the solutions to operations with integers modeling with the number line and two-color counters and then explain why their model correctly relates to the mathematical solution.

b) Students construct a real world situation in which multiplication/division of positive and negative integers occurs. (Examples: distance, credits and debits, or using the Celsius/Fahrenheit formula with negative numbers present). Students will swap their constructed scenario with a partner to solve, using their understanding of multiplying/dividing integers.

Lesson 2: Operations with rational numbers

Given understanding of the procedures of multiplication and division and a sense of value to negative and positive integers (Unit 1); operations with all rational numbers will be expanded upon Lesson 1 with students using tools and estimation to express their understanding (start first with only multiplying/dividing: fractions with fractions or decimals with decimals- and revisit in Lesson 5).

a) Through use of number line and manipulatives, students will demonstrate the solutions to operations with all rational numbers. With number line and ruler activities students can gain insight to working with non-integer positive/negative situations. This is an opportunity for the students to be precise in their solutions and explain why the solution makes sense.

b) Students will continue with the above activity, however, students can use their reasoning stills to estimate solutions prior to using tools and make arguments why their estimate is viable.

c) Students should be able to use multiplication and division rational number operations to solve real world problems. Here students can be given the example of rate and above/below sea level. Given the rate of erosion (using decimals and fractions in situation) how many feet would the area be below sea level after x number of years. In reverse, students can find the rate by being given the number of years and the amount of erosion that has occurred (again, don't mixed decimals and fractions until

Lesson 5). For example students can be given negative or positive quantities and then multiply and divide them into groups. Then explain the procedure they used to solve.

Lesson 3: Math Properties and Rational Numbers

Students will use their newfound understanding of multiplication/division with rational numbers to work with the properties of math. Start first with only operations of fractions with fractions or decimals with decimals- and revisit in Lesson 5).

a) Review math properties and their usage. Expanding upon prior knowledge to include variables, integers, fractions, and decimals (later students will need to be able to work with positive and negative rational coefficients). Given equations that are simplified for the student, in margins student writes the mathematics property that gives validity to the equation to be simplified in that fashion. Given a property and incorrect interpretation of the property in its simplification, student can explain why interpretation is incorrect and give counterexamples demonstrating why invalid.

b) Use mathematical properties to solve real world situations, such as writing algebraic equations with all forms of rational numbers to represent a situation and then using mathematical properties to manipulate the equation. Or given a real world situation where a negative/positive fraction or decimal is present student can use mental math techniques to come up with solution using distributive property and then explain why they used the numbers he/she did.

Lesson 4: Converting fractions to decimals using long division

In order to be able to practically use their understanding of operations with rational numbers students need to know what to do when presented with situations where there are rational numbers in different forms. Therefore students will need to convert from fractions to decimals.

a) Per Common Core Standards 7.NS.2, students will first use the procedure of long division to do conversion of fractions to decimals, students may be given situations where they need to compile different number of items into a given number of boxes needing to find solution of how many items

go into each box, acknowledging that answers may come out as decimals. May use calculator to check work. Then students can look for patterns within the solutions and make generalization. (Example if placing into 8 boxes, all multiples of 8 will be wholes in division. If placing into 9 boxes, other than multiples of 9, answers will be in pattern of increasing by .1111….). Teacher may choose to make place value columns to assure student accuracy.

b) Students will complete conversions of positive and negative fractions to decimals using long division in practice and real world scenarios allowing an opportunity for students to be precise in their calculations. Example would be finding the mean or median of a set of data where the data includes negative rational numbers- be certain to include examples where there are fractions, decimals, and integers. Then students can reason why the median or mean result was what it was from the data. Students can estimate solutions prior to completion and then critique their peers' estimation.

Lesson 5: Working with all rational numbers with Multiplication and Division

Here students are going to "put it all together". Use the understanding of multiplying/dividing with negatives, combined with rational number operations, and converting fractions to decimals. Students should then be able to revisit above activities where there are positive and negative rational numbers in all forms.

a) Given abilities to convert fractions, students will now be able to do multiplication and division for all rational numbers with different forms present (e.g. - 4.25 x 2 ¾). See Lesson 2 a and b for ideas, now extending to situations where negative/positive decimals or fractions are present at the same time.

b) Given abilities to convert fractions, students will now revisit using mathematical properties with different forms of rational numbers
Example of Distributive Property 7/8 (-6 + 3.49). See Lesson 3 a, now extending to situations where negative/positive decimals or fractions are present at same time.

Given abilities to convert fractions, students will now revisit solving real world scenarios with all real numbers in different forms present. See Lesson 1 c for ideas, now extending to situations where negative/positive decimals or fractions are present at same time.

Supplemental Materials and Resources

Literature connection:

Accounting Ledger

- Emphasis on numerous quantities so multiplication and division with positives & negatives are prevalent.

Catalogs/Flyers

- Shopping on a budget

Recipes

- With decimal and fractional quantities.

Funny & Fabulous Fraction Stories: 30 Reproducible Math Tales and Problems to Reinforce Important Fraction Skills by Dan Greenberg

Math Doesn't Suck: How to Survive Middle School Math Without Losing Your Mind or Breaking a Nail by Danica McKellar : covers middle school math topics: factors and multiples, fractions, decimals, percents, word problems, and a little algebra.

Kiss My Math by Danica McKellar : integers & negative numbers, variables and working with expressions, solving (linear) equations, word problems, intro to inequalities, exponents, and an intro to functions and graphing lines.

Interdisciplinary connections:

Science

- The charges of numerous atoms within a compound
- Elevations above and below sea level
- Finding rate of increase or decrease in sea level given distance and time or finding the distance of decrease or increase in sea level given the time and rate.

Social Studies/Geography

- Map building with fractional/decimal values for distances
- Using election data to find number of voters in certain voting blocks/demographics

National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

Online Lessons

- Multiplying Rational Numbers (Lesson with worksheets)
  http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module2-3.html
- Dividing Rational Numbers (Lesson with worksheets)
  http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module2-4.html

- Multiplying and Dividing Integers (Online Lesson and Practice)
  http://www.math.com/school/subject1/lessons/S1U1L12GL.html
- Multiplying and Dividing Integers (Lesson with worksheets)
  http://www.uen.org/Lessonplan/preview.cgi?LPid=23480

Worksheets
- Multiply Integers & Divide Integers Lessons and Practice
  http://teachersites.schoolworld.com/webpages/TAhrens/files/Multiplying%20and%20Dividing%20Integers.pdf
- Multiply and Divide Integers Practice
  http://www.beavertonmath.com/Extended%20Algebra%201/Unit%202%20-%20Integers/IntegersWS6.pdf
- Integer Multiplication and Division Worksheet
  http://mscoxata.wikispaces.com/file/view/Integer+Multiplication.pdf

Videos
- Multiplying and Dividing Integers A (many other videos in menu at right)
  http://www.schooltube.com/video/029f6474bcf84de5a0e4/

SMART Board Lessons
- Integers – Students will be able to learn how to use integers with addition, subtraction, multiplication and division.
  http://exchange.smarttech.com/details.html?id=6ff222a2-413a-4832-9859-4f17e82389a7
- Addition, Subtraction, Multiplication, and Division of Integers – This lesson activity provides practice of the rules for the basic operations with integers.
  http://exchange.smarttech.com/details.html?id=fc8465eb-246c-4c6f-9153-d0c3b681ed30
- And many more: HERE!

Online Interactive Activities & Games
- IXL (Grade 7) – Skills E.6-E.8
- Integers Jeopardy
- MathCar Racing Game
- Integer Warp
- Math Fighter: Integer Operations
- Integers: Multiplication Blocks
- Integers Division
- Integer Games

Tools/Manipulatives
Two color counters Calculator Rulers Playing Cards Graph Paper Number lines

Suggested Formative Assessment Practices/Processes
Teacher created exit slips, teacher created quizzes Think-Pair-Share Agreement Circles Fist to Five: Have students self-assess how well they understand the concept with a show of fingers (from 0-5, 0 = no understanding, up to 5, which = a complete understanding).

Differentiation and Accommodations
understanding. Green cup = no problem, I understand what to do; Yellow cup= I need some assistance but it is not stopping me; Red cup= I need help • Provide graphic organizers • Provide additional examples and opportunities for repetition • Provide tutoring opportunities • Provide retesting opportunities after remediation (up to teacher and district discretion) • Teach for mastery not test • Teaching concepts in different modalities • Adjust homework assignments