Grade/Subject	Grade 6/ Mathematics
Unit Title	Unit 5: Applications of Geometry
Overview of Unit	Students will solve real-world and mathematical problems involving area, surface area and volume. Nets will be utilized as a tool, which may be a significant 6th grade-teaching shift.
Pacing	22 days

Background Information For The Teacher
The Applications of Geometry unit requires the sixth grade teacher to have a keen understanding of nets and their composition and decomposition utilized to find surface area. (Nets are utilized extensively for surface area. Although nets may have been used previously, they must be a key component in student understanding of composition/decomposition of figures in relation to surface area.) Additionally, it will be very beneficial for teachers to use manipulatives, both concrete and virtual, to facilitate student learning for a deeper understanding of area, volume and its real world application. While three-dimensional figures are utilized in fifth grade to calculate volume, they are explored in greater depth in sixth grade. In fourth grade, students will have: • applied the area and perimeter formulas for rectangles in real world and mathematical problems. *(This is not continued in fifth grade.)* In fifth grade, students will have: • used a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines creating a point of origin, • understood that an ordered pair is formulated by how far to travel from the origin, • represented real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, • interpreted coordinate values of points, • understood that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category (for example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles), • classified two-dimensional figures in a hierarchy based on properties.

- recognized volume as an attribute of solid figures and understood concepts of volume measurement; measured volumes by counting unit cubes and related volume to the operations of multiplication and addition and solved real world and mathematical problems involving volume.
- found the area of a rectangle with fractional side lengths and showed that the area is the same as would be found by multiplying the side lengths,
- multiplied fractional side lengths to find areas of rectangles,

In sixth grade, students will:

- find the area of polygons by composing or decomposing the polygon into other shapes,
- draw polygons in the coordinate plane,
- use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate,
- represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures,
- apply all of the above techniques in the context of solving real-world and mathematical problems.
- find volume of rectangular prisms using fractional side lengths and relate this to the formulas for volume ($V = l \cdot w \cdot h$ and $V = B \cdot h$).
- reasoning about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths.

Students will prepare for their work in Grade 8 with transformations by working with polygons in the coordinate plane in Grade 6. Reasoning about relationships in their work on surface area, composing and decomposing shapes, and finding distance on the coordinate plane using endpoint coordinates for horizontal, and vertical lines prepare them for Grade 7 relationships.

Essential Questions (and Corresponding Big Ideas)
How is mathematics used to describe space? In what ways can this be done? o Mathematics can be used to measure, model, and calculate area, volume, and surface area of given polygons, as well as the lengths of segments of a polygon in a coordinate plane. Different methods help us to understand and solve real world mathematics problems.

Core Content Standards	Explanations and Examples
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. This standard concentrates on whole number exponents with a focus on understanding the meaning of exponents and exponential notation such as $3^2 = 3 \times 3$. Students find the value of an expression using exponential notation such as $4^3 = 64$. Students write and evaluate numerical expressions such as: $5 + 2^4 * 6$. What the **teacher** does: • Plan experiences for students to investigate that an exponent is notation representing repeated multiplication such as 10 * 10 * 10 as 10^3. The exponent is the number that tells how many factors of 10 there are. The expression 10^3 is called the exponential expression. Exponential notation was developed to write repeated multiplication more efficiently. • Use patterns to discover why for example 2^0 is one and 35^0 is one. • Allow students to use manipulatives to show $3^2 = 9$ square units. Ask students to model the meaning of $3^3 = 27$ cubic units with manipulatives. • Expand to problems such as $5 + 2^4$ and $7 * 4^3$.	6.EE.1. Examples: • Write the following as a numerical expressions using exponential notation. o The area of a square with a side length of 8 m (Solution: $8^2 m^2$) o The volume of a cube with a side length of 5 ft: (Solution: 5^3 ft^3) o Yu-Lee has a pair of mice. The mice have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice) • Evaluate: 4^3 (Solution: 64) $5 + 2^4 \bullet 6$ (Solution: 101) $7^2 – 24 \div 3 + 25$ (Solution: 67) 27 + 36 = 9 (3 + 4) 63 = 9 x 7 63 = 63 31 + 80 There are no common factors. I know that because 31 is a prime number, it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80 because 2 x 31 is 62 and 3 x 31 is 93. What the **students** do: • Understand the meaning of exponents and exponential notation such as $3^3 = 3 * 3$. • Use appropriate terminology to explain how to evaluate an expression. • Evaluate numerical expressions containing exponents. • Discover that the base to the zero power is 1. Misconceptions and Common Errors: Some students interpret 3^2 as 3 X 2 = 6. This is a common error. Use a number line representation to model the expression. Also, writing the expanded notation of 3^2 =3 x 3 helps students.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

For Example: use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides = 1/2.

What the **teacher** does:

- Explore with students that letters called variables represent unknown numbers and the same rules that apply in operations with numbers also apply in operations with variables.
- Help students translate verbal expressions into numerical expressions by providing a verbal expression such as "the sum of 6 and 4" and asking them to explore ways to write it with numbers. Encourage students to explore synonyms for operations such as: and, plus, and sum, which can all signify addition.
- Model the notation 6n for 6*n because a number and variable written together means multiply.
- Provide varied practice translating numerical expressions into word from and from a word into variable expressions such as "8 less than 2 times a number is $2x - 8$, "4 times the sum of a number and 2 is $4(x + 2)$," or $2(8 + 7)$ is read as the product of 2 times the quantity or sum of 8+7."
- Help students define parts of an algebraic expression, including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient), as this helps students understand the structure of expressions and explain their process for simplifying expressions. For example: **Terms** are values separated by addition and subtraction such as: $x + 3$ contains 2 terms and $2x - 5$ contains 2 terms.

6.EE.2

• The perimeter of a parallelogram is found using the formula $p = 2l + 2w$. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches.

What the **students** do:

- Recognize that variables represent unknown quantities.
- Translate verbal expressions and numerical expressions into verbal expressions.
- Communicate orally and/or in writing about translating and evaluating variable expressions using precise mathematical language, including variables, coefficients, constant, and term.
- Evaluate expressions for given values of variables using the order of operations when appropriate.

Misconceptions and Common Errors:

Some students misunderstand or incorrectly read expressions. Students often confuse x^3 with 3x. To address this situation, ask students to create a chart with the meaning of x^3 and 3x such as:

- Help students concentrate on evaluating algebraic expressions for a given value of a variable, using the order of operations such as the following: Evaluate the expression $2(x + 3^2)$ when x = 5.

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate

The focal point for Standard 8 is solving problems by graphing points in all four quadrants of the coordinate plane. Students learn that the distance from a point on a coordinate plane to an axis is an absolute value. The coordinate plane is used to represent real-world scenarios.

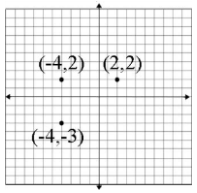
What the **teacher** does:

- Create a coordinate grid on the floor and ask sixth graders to stand on points or place objects on the points. Find the distance between students or objects. Make sure that the two points have either the same first coordinate or the same second coordinate. Lead students to conclude that when you are on the same x or the same y coordinate, you are on the same line.
- Provide a variety of simple word problem experiences for the students to model on a Cartesian plane such as, "My house is at (-3, 5), the shopping mall is at (-4, -2), and the school is at (3, -1). What is the distance between my house and shopping mall? The shopping mall and the school? Explain and show two different ways you used to find the different locations."

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these

x^3 means	3x means
X times x times x X to the third power	3 times x x + x + x

6.NS.8 Example: If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?



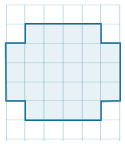
To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is $|-4|$ or 4 units to the left of 0 and 2 is $|2|$ or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis.

techniques in the context of solving real-world and mathematical problems.

Students take triangles and quadrilaterals and form rectangles, or take rectangles and/or other quadrilaterals and decompose them (take apart) into familiar shapes to find the area of the composite shape. A composite shape is a shape formed from other shapes. Students study composite shapes that are unfamiliar and decompose them into familiar shapes such as triangles and rectangles (which they know how to calculate the area of) to find the area. This practice with familiar and irregular composite shapes and decomposition is applied to real-world situations.

What the **teacher** does:

- Make tangram puzzles available for practice in recognizing that several familiar shapes can be put together to compose a rectangle.
- Provide students with a variety of triangles, special quadrilaterals, and polygons and allow them to find the areas of the given shapes. Encourage students to put these shapes together to create new shapes.
- Allow students the opportunity to work backwards by giving them shapes that maybe decomposed into familiar shapes. Begin this as a hands-on experience where students cut apart a polygon into familiar shapes and use what they know about area of the familiar shapes to find the area of the given shape.



Above is a composite shape and may be decomposed into 4 rectangles.

- Ensure students have experiences to write about they are learning. Use exit slips, entrance slips, daily journal entries, and so forth. This allows students to explain how

Students should represent this on the coordinate grid and numerically with an absolute value expression, $|-4| + |2|$.

What the **student** does:

- Understand that a line segment from one coordinate pair to another represents distance.
- Understand that if two coordinates have the same x or y value, they are the same line.
- Understand that the distance from a point on a coordinate plane to an axis is an absolute value.
- Use the coordinate plane to represent the real-world scenarios, such as streets of a map.
- Model solutions to real-world problems on a coordinate plane.

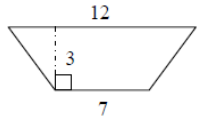
Misconceptions and Common Errors:

Students may have procedural graphing misconceptions and may plot points in spaces rather than intersections. Some sixth graders count intervals on lines rather than x or y axes. Provide hands-on experiences for these learners. Have students plot real objects on a coordinate grid while you observe. Then, have them find the distance between the objects and explain how they found it.

6.G.1. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites. Students can use tools such as the Isometric Drawing Tool on NCTM's Illuminations site to shift, rotate, color, decompose and view figures in 2D or 3D (http://illuminations.nctm.org/ActivityDetail.aspx?ID=125)

Examples:

- Find the area of a triangle with a base length of three units and a height of four units.
- Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



- A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

decomposing a polygon into familiar shapes such as triangles and rectangles can help find the area of the polygon.

- Model using appropriate mathematical vocabulary.
- Use a variety of mathematical and real-world examples of composite figures for students to apply what they are leaning about decomposing figures and area of composite figures.

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

With this standard students build on their background knowledge of volume of right rectangular prisms with whole number dimensions by using manipulatives to determine the volume of a right rectangular prism with fractional side lengths.

Students relate this experience to the formulas for volume *(V=lwh and V = bh)* and find that their experience of counting the unit cubes yields the same result as using the formulas. Students then solve real-world and mathematical problems by applying volume formulas appropriately.

- The area of the rectangular school garden is 24 square units. The length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?

- The sixth grade class at Hernandez School is building a giant wooden H for their school. The H will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

  o How large will the H be if measured in square feet?

  o The truck that will be used to bring the wood from the lumber yard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?

What the **students** do:

- Reason that familiar shapes can be put together to create composite shapes whose area is equal to that of the sum of the areas of the joining shapes.
- Model composite shapes and their decomposition with hands-on materials such as attribute shapes and tangrams.
- Compare decompositions among classmates to see that a shape can be decomposed more than one way.
- Gain confidence in decomposing composite shapes into familiar shapes and use the areas of the familiar shapes to find the area of the composite shapes.
- Generalize how the area of a shape is the sum of the areas of the shapes that make up the composite shape. Write the generalizations to clarify understandings.
- Solve problems from the real-world using composite figures to model real-world examples such as the size of a lake or a crater on the moon.

Misconceptions and Common Errors:
Students who have difficulty performing more than two steps in solving a problem may have difficulty finding the area of the composite figures even after decomposing them. These students benefit from writing the areas of the joined shapes directly in the composite figure to help keep track of the parts. Students can also color code the decomposition.

What the **teacher** does:

- Provide students with the opportunity to fill a right rectangular prism (a box, a folded net, etc.) with cubes. The edges of the cubes should represent a fractional length such as ½ of a unit. If cubes with fractional length edges are not available, use centimeter cubes (or other unit cubes) naming the length of the edges as ½ units. At least one of the edges of the box needs to be a fractional length such as 3 ½ units.
- Pose questions that relate this volume activity to one in which the cubes are whole units. Pose questions about how the number of cubes that filled the box relates to the volume of the box arrived at by using the formula(s).
- Have students write about how this activity compares to one in which whole unit edges are used.
- Prepare real-world problems for students to solve that create a need to calculate volume with fractional units.

6.G.3

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Students plot points in all four quadrants of the coordinate plane. Coordinates are the vertices of polygons. Students connect the points are name the polygons. By giving students coordinates of vertices of the polygons that have the same first or same second coordinate (examples: (3, 4) and
(3, 9) or (7, 6) and (15, 6), students are challenged to find a technique to determine the length of a side of the polygons (subtract same coordinates). Students then apply this knowledge to solve real-world and mathematical problems.

What the **teacher** does:

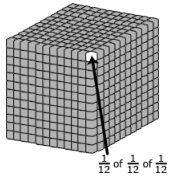
6.G.2

Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

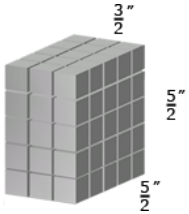
In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

Examples:

• The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ ft ^{3}.



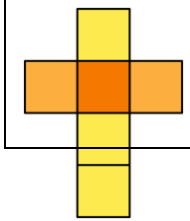
• The models show a rectangular prism with dimensions (3/2), 5/2, and 5/2 inches. Each of the cubic units in the model is $\frac{1}{2}$ in^3**.** Students work with the model to illustrate 3/2 x 5/2 x 5/2 = (3 x 5 x 5) x 1/8. Students reason that a small cube has volume 1/8 because 8 of them fit in a unit cube.



- Provide students with opportunities to draw polygons in the coordinate plane by giving coordinates of triangles, rectangles, and parallelograms.
- Allow students to draw their own polygons and the name the vertices with coordinate points. Students can then trade the coordinates they created with partners and uncover each other's polygons.
- Pose a mathematical problem for students to find the length of a side of a polygon with the same first coordinate (or second coordinate) at the vertices.
- Prepare examples of polygons in Quadrants II, III, and IV so students can apply their knowledge of absolute value.
- Model appropriate use of mathematical vocabulary.
- Promote writing opportunities, including exit and entrance slips and daily journals for students to explain what they are learning. Encourage students to use precise mathematical language in their writing.
- Use real-world and mathematical problems for students to apply this knowledge, such as "What is the area of rectangle whose vertices are (7, 4), (7, 9), (4, 4), (4, 9)?"

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Students begin learning about nets by cutting and folding nets of prisms. Nets are two-dimensional diagrams of three-dimensional shapes that can be folded into the three-dimensional shape. Building on students' previous knowledge of area, students can find the area of the rectangles and triangles that make up given nets. This leads to defining surface area as the sum of the area of the faces of the three-dimensional figure. Once students understand this concept, they solve real-world and mathematical problems involving surface area.



What the **students** do:

- Model volume by filling a rectangle prism with unit cubes of fractional length and use the model to determine volume of the prism.
- Discover that filling a solid with cubes and counting them gives the same result as using the volume formula.
- Reason that finding volume is the same process and uses the same formulas whether the edge lengths are whole units or fractional units.
- Solve real-world volume problems where one of the edge lengths is a fractional unit.

Misconceptions and Common Errors:

Students who understand the relationship between volume as filling of a space with cubes and the volume formula but, due to weak fractional computation skills, may still produce incorrect responses. Provide additional opportunities for these students to improve their computational fluency. Technology offers many solutions for improving computational fluency.

6.G.3

Example:

• On a map, the library is located at (-2, 2), the city hall building is located at (0, 2), and the high school is located at (0, 0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

- o What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
- o What shape is formed by connecting the three locations? The city council is planning to place a city park in this area. How large is the area of the planned park?

What the **students** do:

- Draw polygons in the coordinate plane with attention to naming the vertices with coordinate points.
- Discover how to find the length of sides of polygons using the coordinates of the vertices having the same first coordinate (or second coordinate) and generalize a technique.
- Reflect in wiring on the generalizations, explaining what they learned about how to determine length of a line segment. This can be writing about how they solved, a real-world problem where this generalization was applied.

What the **teacher** does:

- Plan opportunities to familiarize students with nets of prisms. Examples include cutting out nets and folding them into three-dimensional figures, drawing nets for three-dimensional figures, finding the different nets that can fold to make a cube, drawing three-dimensional figures on isometric paper and labeling the faces, and so on.
- Promote problem-solving experiences using nets such as, "How many different nets can be made to fold up to a cube?"
- Lead students to find the faces of a net. Model folding the net back to its three-dimensional figure and use this to define surface area. Shade the faces of the net different colors to help students see the relationship of the individual areas to the total surface area.
- Encourage students to find examples where the dimension of the faces repeat in a net. For example, in a rectangular prism, the opposite faces have the same dimensions.
- Pose prompts so students can write about surface area. An example of a prompt is the following: Peter has no idea what surface area is. Explain to him in word and pictures.
- Encourage students to see correct mathematical vocabulary when speaking and writing about surface area.
- Facilitate a class discussion for students to share what they discovered about surface area and challenge on another's idea.
- Provide real-world and mathematical problems for students to apply what they have learned about surface area. Include diagrams of the shapes that go along with the problems.

Misconceptions and Common Errors:

Students who confuse knowing which coordinates to subtract may have memorized an algorithm for finding the distance (length of side) without understanding how to use the coordinates on the plane. To address this, provide additional experiences drawing polygons and explaining (orally and in writing) how to find the length of a side with same first (and the same second) coordinates. Communication helps students clarify this understanding.

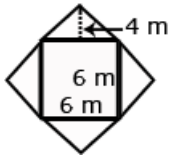
To prevent the misconception that coordinates only appear in the first quadrant, it is important to use coordinate points in all four quadrants. This means that students will need to have previous experience with negative integers so they can find points such as (-3, -2).

6.G.4 Students construct models and nets of three dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205).

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Examples:

	• Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not? • Create the net for a given prism or pyramid, and then use the net to calculate the surface area.   What the **students** do: • Visualize how nets relate to three-dimensional figures. • Use a model to determine surface area. • Understand how area of two-dimensional figures relates to surface area of three-dimensional figures. • Compare and contrast area and surface area in writing using mathematical vocabulary. • Gain confidence as students share ideas in a supportive forum. • Solve real-world and mathematical problems involving surface area. Misconceptions and Common Errors: Some students who have difficulty with multi-step problem solving may also have difficulty with surface area because these are six areas to calculate and add together for a prism. To help, shade the faces different colors and write the areas of the faces directly on the net or on the diagram of the three-dimensional figure. Encouraging students to count the number of faces on the three-dimensional shape before they calculate the surface area may also help account for all the faces. Another common error students make is confusing volume with surface area in problems where students must determine which concept should be applied. After students have been introduced to both terms, have students make paper foldables to compare the two terms.

Standards for Mathematical Practice	**Explanations and Examples**
Apply and extend previous understandings of arithmetic to algebraic expressions. **6.EE.1, 6.EE.2** The focus of this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as $3^3 = 27$, using the appropriate terminology to explain how to evaluate an expression. Students are applying properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation. **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.** **MP6. Attend to precision.**	Sixth graders decontextualize to manipulate symbolic representations by applying properties of operations. Students model real-world scenarios with equations and expressions. Students communicate precisely with others and use clear mathematical language when describing expressions.
Solve real-world and mathematical problems involving area, surface area, and volume. **6.G.1, 6.G.2, 6.G.3, 6.G.4** The cluster focuses on area, volume, and surface area. Students use knowledge and skills to solve real-world and mathematical problems and apply the concepts by manipulating nets, cubes, and other real-world materials. **MP4. Model with mathematics.**	Students apply what they learn about area, surface area, and volume to real-world and mathematical problems.

<table>
<tr><th colspan="2">K-U-D</th></tr>
<tr><th>KNOW
Facts, formulas, information, vocabulary</th><th>DO
Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)</th></tr>
<tr><td>• Strategies for finding the area of polygons, including triangles and special quadrilaterals
• Area is measured in square units
• Coordinate planes can be used to describe side lengths of polygons
• Nets represent three-dimensional figures in two-dimensional space
• Surface area is covering all faces of a three-dimensional shape with square units
Volume of a right rectangular prism with fractional edge lengths</td><td>• FIND/DETERMINE area of polygons
• JUSTIFY formulas for areas of triangles and parallelograms
• COMPOSE polygons into rectangles
• DECOMPOSE polygons into triangles and other shapes
• DRAW polygons on a coordinate plane
• USE coordinates to find side lengths
• REPRESENT three-dimensional figures using nets
• USE nets to find surface area
• SOLVE problems with and without context
• APPLY problems with and without context
• FIND volume of right rectangular prism using the formulas V = l*w*h and V = B*h (when B is l*w)</td></tr>
<tr><th colspan="2">UNDERSTAND
Big ideas, generalizations, principles, concepts, ideas that transfer across situations</th></tr>
<tr><td colspan="2">Area and volume developed and justified in different ways (such as decomposing two- and three-dimensional figures into smaller component figures) can help in understanding and solving real world and mathematical problems.

One can find the value of a whole, by combining the values of its parts.</td></tr>
<tr><th colspan="2">Common Student Misconceptions for this Unit</th></tr>
<tr><td colspan="2">• Students may believe the orientation of a figure changes the figure. For example, a square rotated 45° may be called a diamond instead of a square.
• When finding the area of a triangle some students may forget to multiply by half or divide by 2. They need to be reminded to</td></tr>
</table>

think about the relationship between a rectangle and a triangle.

- Students may not realize that multiplying by ½ is the same as dividing by 2 or that taking half the base or the height and then multiplying by the other factor will result in the same answer.
- Students may not (or forget to) add the separate area calculations of a figure to find its total surface area.
- Students may have a difficult time visualizing a two-dimensional net as a three-dimensional figure. This may lead to leaving a face or faces out of the surface area calculation.
- Students may find difficulty decomposing irregular figures into appropriate rectangles and triangles.
- Students may ignore or miscalculate missing lengths on irregular figures before calculating total area.
- Students may confuse "B" with "b" in regard to base in calculation. ("B" refers to the base of a 3-dimensional shape [length multiplied by width] where "b" refers to the base of 2-dimensional shape.)

Unit Assessment/Performance Task	DOK
Unit 5 Test Unit 5 Performance Task "In the Playground"	

Vocabulary
Area
Base
Compose
Congruent Figures
Coordinate
Cube
Decompose
Diagonal

Dodecahedron Heptagon
Edge
Face(s)
Grid
Grid/Plane
Hexagon
Icosahedron
Net
Octagon
Octahedron
Parallel Lines
Parallelogram
Pentagon
Perpendicular (line)
Plane
Platonic solids
Plot
Polygons
Polyhedron
Prism
Quadrilateral
Rectangle
Rhombus
Right Rectangle Prism
Right Triangle
Similar Figures
Square

Square Units
Surface area
Tetrahedron
Trapezoid
Triangle
Unit Cube
Vertex
Volume

Key Learning Activities/Possible Lesson Focuses (order may vary)

The following activities are broken into "lessons," even though each may take more or less than one class period depending on school schedule.

These are ideas for lessons.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Lesson 1: Area of Triangles

To begin Lesson 1, review the area formula for rectangles. Display a two dimensional rectangle packed with unit cubes. Diagonally cut the rectangle into halves. Lightly shade one-half of the diagram. Allow students some time to calculate the number of cubes within each half. This activity will begin the discussion of how right triangles can be composed into a rectangle and a rectangle can be decomposed into triangles. Also, the formula for finding the area of a triangle can be introduced here (A = ½ b x h).

Additionally, having students use standard 8 ½ x 11 sheets of paper to explore the decomposition of a rectangle into two triangles is helpful as an easy manipulative. Students can

fold or cut a piece of paper and then compare it to another full sheet to show that the two decomposed figures total the whole figure.

Terminology communication is important in this lesson (and unit). Teachers can also emphasize that dividing a whole into two separate parts is the same as multiplying by one-half. ("Dividing in half" means "dividing by two". "Cutting in half" means multiplying by half".)

Suggested Resources:

- Mathematics Station Activities for CC: Appropriate Units of Measurement. Pg. 91
- Teaching CC math standards with hands-on activities: It's Half activity pg. 54
- The following website will allow students the opportunity to calculate area of triangles presented on a coordinate grid. While it is not composing or decomposing to or from other polygons, it will connect side joints of a polygon presented on a plane which is presented in Lesson 3.

http://www.shodor.org/interactivate/activities/TriangleExplorer/

Lesson 2: Area of Special Quadrilaterals and Polygons

Building from Lesson 1, a parallelogram can be displayed on a piece of grid paper with its height displayed appropriately. Students can "cut" the right triangle (created with the height segment and part of the parallelogram base) and place in on the opposite side of the parallelogram. This will create a rectangle. From here, the area formula for a rectangle (A = b x h) can be reviewed and related to parallelograms. As an extension, a display of a parallelogram can be cut diagonally into two triangles. The triangle formula can be applied to each, and combined to determine the area of the whole parallelogram.

Other special quadrilaterals can be introduced and manipulated in similar fashions. Trapezoids can be decomposed into triangle(s) and rectangle(s). "L" and "H" shaped right-angled polygons

can be decomposed into rectangles. Additional alphabetical letters can be used as models for students to find the area of irregular polygons.

Suggested Resources:
The following website is an introduction to activities that allow users to manipulate polygons and see the relationship between dimensions and area. There are several links in the website itself to additional activities.
http://illuminations.nctm.org/LessonDetail.aspx?id=U160

Lesson 3: Polygons in the Coordinate Plane (Second half of Section 9-1 and Section 9-2.)
Making a link between the use of a Geoboard and a coordinate grid can help students describe the location of points on a plane. With a Geoboard, students can make a rectangle with a rubber band around the pegs. Have students identify the lower left peg as a coordinate (1, 1). Ask students to share their upper left, upper right and lower right coordinate points. Ask students to determine who has the largest (and smallest, similar, congruent, etc.) rectangle display on their Geoboard. Students can then make and identify other polygons with the Geoboards. Additionally, the length of the sides of the created polygons can be determined by counting pegs, which can then be connected to a coordinate plane.

Teachers can review plotting points on a coordinate grid (a 5th grade standard). Activities should progress to plotting points and connecting plots to create polygons. Areas of these created polygons can then be calculated.

Suggested Resources:
Virtual Geo-board: www.mathlearningcenter.org/web-apps/geoboard/

Lesson 4: Nets and Surface Area
Students can bring in an empty cardboard box from home (shoe box, cereal box, snack food box, etc.) that may be cut up. Have students cut the box along the edges without cutting any of the

faces off. Have the students lay the box flat. Discuss with students that each box was a rectangular prism and now is a net with 6 faces. The surface area can be identified as the "plain" or undecorated side of the net.

Additionally, to prepare for the Performance Task, discussing the area of each face of a rectangular prism that does not have a lid (such as an empty aquarium) will be helpful.

Suggested Resources:

- Teaching CC math standards with hands-on activities: Finding the surface area of nets. Pg. 63

Mathematics Station A

Lesson 5: Volume of Right Rectangular Prism (3 days)

Students will find volume of right rectangular prism using the formulas V = l·w·h and V = B·h (when B is l·w). Draw a 2-dimensional rectangle on the board. Ask students how to find its area. Now, draw a 3-dimensional rectangular prism on the board. Ask students how this figure's area would differ from the first shape.

Goals

- Find the volume of a right rectangular prism with fractional edge lengths by packing it with cubes of an appropriate unit fraction edge length.
- Show that volume is the same as multiplying the edge length of a rectangular prism
- Apply the formulas to find volume of right rectangular prisms with fractional edge lengths in context of solving real-world and mathematical problems.

Suggested Activities

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

Students will individually explore a rectangular prism to derive its volume first using units then a formula.

- http://www.learnnc.org/lp/pages/3487
  Students will work in small groups to determine volume of an actual rectangular prism (single serving size cereal box).
- "It's Voluminous" Activity
  Working in small groups, students can fill a rectangular prism with cubes by making a sketch and then finding the volume of the prism by using formulas.
- "Let's Make a Stack" Activity
  Students will work cooperatively to solve problems detailing decimal computation and volume.
  (See Math06_Unit02_SuggestedActivity_LetsMakeAStack for activity)

Resources

- Crosswalk Coach Grade 6, Domain 4 Lesson 31
- Teaching the Common Core Math Standards, Geometry: 6.G.2

- Activities for CC: Visualizing Solid Figures. Pg. 98

The following websites allow students to manipulate figures virtually.
http://nlvm.usu.edu/en/nav/grade_g_3.html
http://nlvm.usu.edu/en/nav/search.html?qt=geoboards
http://www.senteacher.org/wk/3dshape.php

Supplemental Materials and Resources

- Connected Math 2, Pearson/Prentice Hall

- Crosswalk Coach Mathematics Grade 6, Triumph
- Teaching the Common Core Math Standards, Jossey-Bass
- Big Ideas Math, A Common Core Curriculum, Larsen/Boswell

Literature Connections
Counting on Frank by Rod Clements
Grandfather Tang's Story by Ann Tompert
The Fly on the Ceiling by Julie Glass
The Greedy Triangle by Marilyn Burns

Interdisciplinary Connections

Science (area and volume are used in scientific calculations)
Art (polygons, prisms, nets)

Tools/Manipulatives

Calculators
Measurement Tools (rulers, tape measures, etc.)
Universally sized counters (such as single base ten block counters)
3-dimensional rectangular prisms (such as empty single-serve breakfast cereal boxes)
Graph paper
Coordinate Grid paper
Empty cardboard boxes
Inch cube or centimeter cubes
Rulers
Tangrams

Suggested Formative Assessment Practices/Processes

Teacher created exit slips, teacher created quizzes

Differentiation and Accommodations

During lessons and activities, students will use 3 colored cups to display their understanding of the concept being worked on. A Green cup states the group/individual is working well and they are understanding the concept. A Yellow cup states the group/individual is working but has a question. A Red cup states the group/individual does not understand the concept and needs immediate attention.

- Provide graphic organizers
- Provide additional examples and opportunities for repetition
- Provide tutoring opportunities
- Provide retesting opportunities after remediation (up to teacher and district discretion)
- Teach for mastery not test
- Teaching concepts in different modalities
- Adjust homework assignments

Observation and Anecdotal Records

Exit Slip Ideas:

Math06_Unit04_ExitSlipTriangles

Students can show the decomposition of a special polygon to find its area.

Students can identify the net that can be decomposed from a prism and vice versa.

Rating System - Students can rate their understanding of the concept today on a 1, 2, 3 basis.

Students can also be prompted to answer the questions, “What did I learn today?” and “What do I need help with?”