Grade/Subject	Grade 6/ Mathematics
Unit Title	Unit 1: Understanding Positive and Negative Numbers
Overview of Unit	In this unit students will apply and extend previous understanding of numbers to the system of rational numbers. They will develop an understanding of numbers less than zero and their real-world applications.
Pacing	21 days

Background Information For The Teacher
In previous grades students used number lines as a tool and in fifth grade, students learned to graph points on the coordinate plane in the first quadrant to solve real-world and mathematical problems. In this unit, students will extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane. Prior to the Common Core Standards, sixth grade students were responsible for operating (addition and subtraction) with integers. There has been a shift in learning where students are now developing a strong understanding of positive and negative numbers and how they are used in the real-world. All activities used throughout this unit should be related to the real world in order to help students model with mathematics. A student-centered approach will foster students' understanding of the concepts. Through student discourse, and justification of their answers, students should regularly be required to construct viable arguments and critique the reasoning of others in order to further develop their understanding.

Essential Questions (and Corresponding Big Ideas)

How do I plot and order whole numbers, fractions, and decimals on a number line?

- Use inequalities to compare two numbers in the same form or different forms.
- Proper fractions are between 0 and 1 on the number line.

Why are there negative numbers?

- When zero is used as the origin it is useful in describing a unit of space.
- To show a mathematical relation of loss.

When will I ever need to use negative numbers?

- Negative numbers can be used to represent quantities less than zero or quantities with an associated direction.

When will I ever use greatest common factor and least common multiple?

- The greatest common factor and least common multiple are used when operating with fractions.

Core Content Standards	Explanations and Examples
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. This standard concentrates on whole number exponents with a focus on understanding the meaning of exponents and exponential notation such as $3^2 = 3 \times 3$. Students find the value of an expression using exponential notation such as $4^3 = 64$. Students write and evaluate numerical expressions such as: $5 + 2^4 * 6$. What the **teacher** does: • Plan experiences for students to investigate that an exponent is a notation representing repeated multiplication such as 10 * 10 * 10 as 10^3. The exponent is the number that tells how many factors of 10 there are. The expression 10^3 is called the exponential expression. Exponential notation was developed to write repeated multiplication more efficiently. • Use patterns to discover why for example 2^0 is one and 35^0 is one. • Allow students to use manipulatives to show $3^2 = 9$ square units. Ask students to model the meaning of $3^3 = 27$ cubic	6.EE.1. Examples: • Write the following as a numerical expressions using exponential notation. o The area of a square with a side length of 8 m (Solution: $8^2 m^2$) o The volume of a cube with a side length of 5 ft: (Solution: 5^3 ft^3) o Yu-Lee has a pair of mice. The mice have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice) • Evaluate: o 4^3 (Solution: 64) o $5 + 2^4 \bullet 6$ (Solution: 101) $7^2 – 24 \div 3 + 25$ (Solution: 67) What the **students** do: • Understand the meaning of exponents and exponential notation such as $3^3 = 3 * 3$. • Use appropriate terminology to explain how to evaluate an expression. • Evaluate numerical expressions containing exponents. • Discover that nay base to the zero power is 1. Misconceptions and Common Errors:

units with manipulatives.
- Expand to problems such as $5 + 2^4$ and $7 * 4^3$.

6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no *common factor. For example, express 36 + 8 as 4(9+2).*

The emphasis for this standard is finding factors and multiples of a given number(s). Students need to know that numbers being multiplied are the factors, and the product is the multiple. Explore two different methods for factoring.

Introduce the distributive property as an application of factors. When you add two numbers that have a common factor such as 36 and 8, you can remove the greatest common factor, 4, and distribute it to the remaining factors. Such as: 36 + 8 = (4 X 9) + (4 X 2) = 4(9 + 2)

What the **teacher** does:

- Explore finding the greatest common factor (GCF) of two whole numbers less than or equal to 100. The greatest common factor is the largest number that is a factor of two whole numbers. For example, 30 and 18 can be found first by
  - Listing the factors of 30 (1, 2, 3, 5, 6, 10, 15, 30)
  - Listing the factors of 18 (1, 2, 3, 6, 9, 18) and noticing the largest number in each list is the greatest common factor of (6)

  Examine another way for students to find the GCF. List the prime factors of two numbers and then multiply the common factors. For example, 40 = (2X2X2X5) and 16 = (2X2X2X2), and then multiply prime common factors (2X2X2= 8).
- Ensure students understand the process of prime factorization. Prime factorization is finding which prime

Some students interpret 3^2 as 3 X 2 = 6. This is a common error. Use a number line representation to model the expression. Also, writing the expanded notation of
3^2 =3 x 3 helps students.

6.NS.4

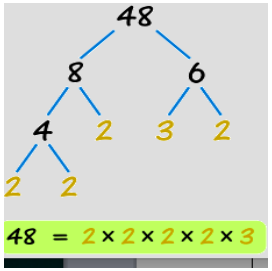
- What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF?
  Solution: $2^2 * 3 = 12$. Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus 2 x 2 x 3 is the greatest common factor.)

- What is the least common multiple (LCM) of 12 and 8? How can you use multiple lists or the prime factorizations to find the LCM?
  Solution: $2^3 * 3 = 24$. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 12 and a multiple of 8. To be a multiple of 12, a number must have 2 factors of 2 and one factor of 3 (2 x 2 x 3). To be a multiple of 8, a number must have 3 factors of 2 (2 x 2 x 2). Thus the least common multiple of 12 and 8 must have 3 factors of 2 and one factor of 3 (2 x 2 x 2 x 3).

- Rewrite 84 + 28 by using the distributive property. Have you divided by the largest common factor? How do you know?
- Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

  - 27 + 36 = 9 (3 + 4)
    63 = 9 x 7

numbers multiply together to make the original number such as the number 24 can be written as a product of prime numbers: 24 = 2 X 2 X 2 X 3. This called the prime factorization of 24. Model prime factorization for numbers 1-100 using factor trees.



- Investigate the distributive property using sums and its use in adding numbers 1-100 with a common factor. For example, 20 + 24 = 4(5 + 6). Use a common factor and the distributive property to find the sum of 36 and 8.
  36 + 8 = 4(9) + 4(2)
  44 = 4 (9 + 2)
  44 = 4(11)
  44 = 44

- Emphasize the least common multiple as it relates to this standard. The least common multiple (LCM) of two numbers is the smallest number (not zero) that is a multiple of both. For example, for 3 and 4, first list the multiples of each to discover the smallest number that is a multiple of both is 12.
  Multiples of 3: 3, 6, 9, 12, 15, 18, 21,...
  Multiples of 4: 4, 8, 12, 16, 20, 14,...
  The LCM of 3 and 4 is 12
- Pose questions such as, "What is the least common multiple (LCM) of 10 and 6? How can you use multiple list or the prime factorizations to find the LCM?
- Focus on the following vocabulary terms: distributive property, factor, greatest common factor (GCF), least common multiple (LCM), multiple, prime factorization, and factor trees.
- Present problem-solving situations to the students to apply their knowledge. A great example is the famous hotdog/bun problem. "Hot dogs come in a package of 8 and buns come in a package of 12. How many packages of

63 = 63

- 31 + 80

There are no common factors. I know that because 31 is a prime number, it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80 because 2 x 31 is 62 and 3 x 31 is 93.

What the **students** do:

- Understand that a factor is a whole number that divides without a remainder into another number.
- Understand that a multiple is a whole number that is a product of the whole number and any other factor.
- Calculate GCF and LCM for given sets of numbers.
- Apply knowledge of factors and multiples to problem solving.
- Communicate, using precise mathematical language, how the distributive property makes use of factors and multiples.

Misconceptions and Common Errors:

Some students may confuse the concepts of factors and multiples. To help with this, use the vocabulary of factors and multiples when working with multiplication and division such as the numbers being multiplied are the factors; the product is the multiple. Paper foldables with vocabulary definitions or mathematical games may also help students practice confusing vocabulary terms.

hot dogs and packages of buns would you need to purchase to have an equal number of hot dogs and buns?

- Ensure students have opportunities to talk with the teacher and each other to make sense of what they are learning about common factors and common multiples using precise mathematical vocabulary.

6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g.,temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

In this standard, students investigate positive and negative number (integers) in real-world scenarios as being opposite values or opposite directions such as 10^0 below zero (-10) and 10^0 above zero (+10). They use vertical and horizontal number lines to show all rational numbers and must explain that the meaning of zero is determined by the real-world context.

What the **teacher** does:

- Explore with multiple examples and experiences using positive and negative integers to represent real-world situations such as a bank account with credits and debits, temperature, and above and below sea level.
- Investigate the use of both verbal and horizontal number lines to illustrate examples such as, “Our football team lost 7 yards on the first down.” Or, “It is freezing outside today and is 10 degrees below zero.” Or, “The bank statement for the middle school football team has a balance of $4,026. The coach bought new equipment for the team for a total of $4,400. How much money should the coach deposit into the football account in order to stop the account from being overdrawn?”
- Have students create their own examples to show on their number lines and explain the meaning of 0 in each situation.
- Pose questions such as, “When you look at the number lien, what do you notice about the location of the negative

What the **students** do:

- Understand that zero represents a position on the number line.

numbers?" which will lead students to discover that all negative numbers are less than zero.

6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

The heart of this standard focuses on previous understanding with the use of both horizontal and vertical number lines. Students extend graphing points and reflecting across zero on a number line to graphing and reflecting points across axes on a coordinate grid. They identify and plot coordinates in all four quadrants of a coordinate plane.

What the **teacher** does:

- Discover that every negative integer is less than zero.
- Understand the meaning of zero is determined by the real-world context. For example, on a Celsius thermometer, everything below zero is negative, and everything above zero is positive.
- Represent real-world scenarios such as bank account balances, temperature, and sea level with integers.
- Use precise mathematical vocabulary to discuss positive and negative numbers.

Misconceptions and Common Errors:

Some sixth graders may believe the greater the magnitude of a negative number, the greater the number. To help with this misconception, continue to use the number line. Have the students trace a horizontal number line with a finger starting at a positive number such as 10 and moving left one number at a time. Ask the student each time the finger moves one number if the number is getting larger or smaller. Continue across 0. By then, a pattern of numbers getting smaller as you move left on the number line should be established.

6.NS.6. Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids.

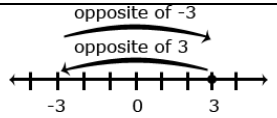
- Facilitate a discussion around a number line that allows students to discover that a number and its opposite are equidistant from zero. The opposite sign (-) shifts the number to the opposite side of 0 such as -5 is stated as "the opposite of 5," which is negative 5. Remind students that zero is its own opposite, and the opposite of the opposite of a number is the number itself, such as –(-5) = 5.
- Plan an activity for students to place numbers on a number line such as -3.5, 1, 4.2, -5, 9/2 and justify their order.

  Then, students can make statements about the relationships between the numbers.

- Help students relate graphing points and reflecting across zero on a number line to graphing and reflecting points across axes on a coordinate grid. As the x-axis and y-axis are extended to include negatives, help students recognize the point where the x-axis and y-axis intersect as the origin, to identify the four quadrants, and to identify the quadrant for an ordered pair based on the signs of the coordinates, such as Quadrant I (+,+), Quadrant II(-,+), Quadrant III(-,-), and Quadrant IV (+,-)
- Plan an activity for students to discover the relationship between two ordered pairs differing only by signs as reflections across one or both axes. Have them plot the ordered pairs
  (-4,5) and (-4,-5), to discover the y-coordinates differ only by signs, which represents a reflection across the x-axis. Students should explain why a change in the x-coordinates from (-4,5) to (4,5) represents a reflection across they y-axis. When the signs of both coordinates change (4,-5) changes to (-4,5), the ordered pair has been reflected across both axes.

- Ensure students have opportunities to talk with the teacher and each other to make sense of what they are learning about recognizing opposite signs of numbers; recognizing that when two ordered pairs differ only by signs the location of the points are related by reflections across one or both axes; finding and positioning integers and other rational numbers on a horizontal or vertical number line diagram; and finding and positioning pairs of integers and other rational numbers on a coordinate plane.

6.NS.7. Understand ordering and absolute value of rational numbers.



Example:

• Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point?

$\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ (0.25, -0.75)

What the **students** do:

- Understand the meaning of the term opposite and plot opposites on a numbered line.
- Reason that the opposite of the opposite of the number is the number itself (e.g. –(-3) and zero is its own opposite.
- Describe quantities having opposite values.
- Use vertical and horizontal number lines to show integers.
- Plot integers on the number line and coordinates in all four quadrants of a coordinate plane.
- Understand that the signs of numbers in ordered pairs represent a singular location on the coordinate plane.
- Understand that an ordered pair is composed of two parts: The first coordinate refers to the x-axis, and the second coordinate refers to the y-axis.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3 C > – 7 C to express the fact that –3 C is warmer than – 7 C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write /-30/ = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than – 30 dollars represents a debt greater than 30 dollars. This standard focuses on understanding the ordering and the absolute value of rational numbers. Students explore the meaning of absolute value as the distance from zero on a number line. They learn that the value of -5 is less than -3 and that with negative numbers, as the absolute value increases, the value of the number decreases.	• Recognize the signs of all ordered pairs on the coordinate plane: Quadrant I (+,+, Quadrant II (-,+), Quadrant III (-,-), Quadrant IV (+,-). • Understand that changing the sign of one or both numbers in the ordered pair will create a reflection of the point. • Find reflection points across axes. <u>Misconceptions and Common Errors:</u> Some sixth graders do not understand that negative signs change a number to the same distance on the opposite side of 0. Use a tool such as a ruler to measure the distance to prove this is true. Some students confuse quadrant labels I through IV going counterclockwise. When introducing the quadrants, have students write the quadrant numbers in the quadrants to help them remember. Some learners may confuse (3, 2) and (-3, 2), thinking both ordered pairs look the same. Using paper folding or mirrors may help the students understand the connection between signs on coordinates and their reflections across the axes.

Students interpret the absolute value in a real-world scenario refers to magnitude. For example, in the case of a debt of -30 dollars, the absolute value, 30, is the magnitude or size of the debt. Emphasis in this standard is also placed on comparing rational numbers using inequality symbols.

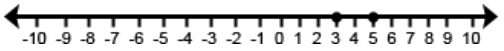
What the **teacher** does:

- Use models such as number lines, thermometers, and checkbooks for profit/loss to focus instruction on ordering and inequalities.
  - Use number line models to order and locate integers and other rational numbers on the number line.
  - Facilitate a discussion that uncovers the fact that as you move left on the number line, numbers get smaller. Use this knowledge to help students write inequalities such as -3<-2.
  - Use thermometers to demonstrate inequalities. Then, provide real-world scenarios to help students make sense of what they are learning, For example. "On Wednesday the temperature was -8 degrees F and on Thursday the temperature was -5 degrees F. Which day was colder? Model it on a horizontal number line, write an inequality, and explain how you know your answer is correct."
  - Use profit/loss models. A positive number corresponds to a profit and a negative number corresponds to a loss. Provide real-world scenarios such as checkbook balances. "My check balance was -$20.40. My friend's checkbook balance was -$8.50. Write an inequality to show the relationship between the checkbook amounts. Who owes more?"
- Have students create and write their own real-life scenarios using two rational numbers and share them with the class. For each scenario, have students write an inequality to represent the situation.
- Lead students to discover the absolute value or magnitude of a rational number as the distance from zero and recognize the symbols.
- Provide real-world scenarios where students use the absolute value of the numbers to answer the questions such as, "Cecily has -30 dollars in her account. What does that mean?" Answer: It means that Cecily is short or owes her bank 30 dollars.
- Ensure that multiple experiences are provided for students to understand the relationships between numbers, absolute value, and statements about order.

6.NS.7. Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers.

In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers.

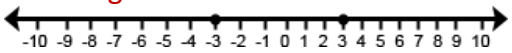
Case 1: Two positive numbers



5 > 3

5 is greater than 3

Case 2: One positive and one negative number



3 > -3

positive 3 is greater than negative 3

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate

The focal point for Standard 8 is solving problems by graphing points in all four quadrants of the coordinate plane. Students learn that the distance from a point on a coordinate plane to an axis is an absolute value. The coordinate plane is used to represent real-world scenarios.

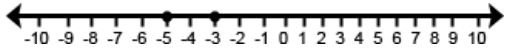
What the **teacher** does:

- Create a coordinate grid on the floor and ask sixth graders to stand on points or place objects on the points. Find the distance between students or objects. Make sure that the two points have either the same first coordinate or he same second coordinate. Lead students to conclude that when you are on the same x or the same y coordinate, you are on the same line.
- Provide a variety of simple word problem experiences for the students to model on a Cartesian plane such as, "My house is at (-3,5), the shopping mall is at (-4,-2), and the school is (3, -1). What is the distance between my house and the shopping mall? The shopping mall and the school? Explain and show two different ways you used to find the different locations."

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate of the same second coordinate. Apply these techniques in the context of solving real-world

negative 3 is less than positive 3

Case 3: Two negative numbers



-3 > -5

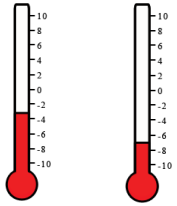
negative 3 is greater than negative 5

negative 5 is less than negative 3

Comparative statements generate informal experience with operations and lay the foundation for formal work with operations on integers in grade 7.

Example:

• One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.

Example:

• The Great Barrier Reef is the world's largest reef system and is located off the coast of Australia. It reaches from the surface of the ocean to a depth of 150 meters. Students could represent this value as less than -150 meters or a depth no greater than 150 meters below sea level.

What the **students** do:

- Order rational numbers on a number line.
- Compare rational numbers using inequality symbols and justify orally and/or in writing the inequality symbol used.

and mathematical problems.

Students plot points in all four quadrants of the coordinate plane. Coordinates are the vertices of polygons. Students connect the points are name the polygons. By giving students coordinates of vertices of the polygons that have the same first or same second coordinate (examples: (3, 4) and
(3, 9) or (7, 6) and (15, 6), students are challenged to find a technique to determine the length of a side of the polygons (subtract same coordinates). Students then apply tis knowledge to solve real-world and mathematical problems.

What the **teacher** does:

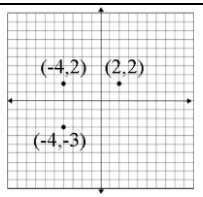
- Provide students with opportunities to draw polygons in the coordinate plane by giving coordinates of triangles, rectangles, and parallelograms.
- Allow students to draw their own polygons and the name the vertices with coordinate points. Students can then trade the coordinates they created with partners and uncover each other's polygons.
- Pose a mathematical problem for students to find the length of a side of a polygon with the same first coordinate (or second coordinate) at the vertices.
- Prepare examples of polygons in Quadrants II, III, and IV so students can apply their knowledge of absolute value.
- Model appropriate use of mathematical vocabulary.
- Promote writing opportunities, including exit and entrance slips and daily journals for students to explain what they are learning. Encourage students to use precise mathematical language in their writing.
- Use real-world and mathematical problems for students to apply this knowledge, such as "What is the area of rectangle whose vertices are (7, 4), (7, 9), (4, 4), (4, 9)?"

- Understand, compare, and interpret rational numbers found in real-world scenarios.
- Discover absolute value of a rational number as its distance from 0 on the number line.
- Model absolute value with number lines. Correctly use absolute value symbols.

Misconceptions and Common Errors:

Common misconceptions occur when students are unable to order rational numbers on the number lien. Some students may incorrectly place -1 ¾ between -1 and 0 instead of between -2 and -1. To address this, have students order the opposites For example, if a student has difficulty placing -1 ¾ on the number line, have the student place + 1 ¾ . Discuss with the students how 1 ¾ came between 1 and 2. Then use that reasoning to help the student place -1 ¾.

6.NS.8 Example: If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?



To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is $|-4|$ or 4 units to the left of 0 and 2 is $|2|$ or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, $|-4| + |2|$.

What the **students** do:

- Understand that a line segment from one coordinate pair to another represents a distance.
- Understand that if two coordinates have the same x or y value, they are on the same line.
- Understand that the distance from a point on a coordinate plane to an axis is an absolute value.
- Use the coordinate plane to represent real-world scenarios. Such as streets of a map.
- Model solutions to real-world problems on a coordinate plane.

Misconceptions and Common Errors:

Students may have procedural graphing misconceptions and may plot points in spaces rather than intersections. Some sixth graders count intervals on lines rather than x or y axes. Provide hands-on experiences for these learners. Have students plot real objects on a coordinate grid while you observe. Then, have them find the distance between the objects and explain how they found it.

6.G.3. Example:

• On a map, the library is located at (-2, 2), the city hall building is located at (0, 2), and the high school is located at (0, 0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

o What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?

o What shape is formed by connecting the three locations? The city council is planning to place a city park in this area. How large is the area of the planned park?

	What the **students** do: • Draw polygons in the coordinate plane with attention to naming the vertices with coordinate points. • Discover how to find the length of sides of polygons using the coordinates of the vertices having the same first coordinate (or second coordinate) and generalize a technique. • Reflect in writing on the generalizations, explaining what they learned about how to determine length of a line segment. This can be writing about how they solved, a real-world problem where this generalization was applied. Misconceptions and Common Errors: Students who confuse knowing which coordinates to subtract may have memorized an algorithm for finding the distance (length of side) without understanding how to use the coordinates on the plane. To address this, provide additional experiences drawing polygons and explaining (orally and in writing) how to find the length of a side with same first (and the same second) coordinates. Communication helps students clarify this understanding. To prevent the misconception that coordinates only appear in the first quadrant, it is important to use coordinate points in all four quadrants. This means that students will need to have previous experience with negative integers so they can find points such as (-3, -2).
Standards for Mathematical Practice	**Explanations and Examples**
Compute fluently with multi-digit numbers and find common factors and multiples. **6.NS.4** Fluency and accuracy with multi-digit addition, subtraction, and division is the focus for this cluster along with a spotlight on greatest common factors and least common multiples. The cluster also builds on previous leaning of the multiplicative structure as well as prime and composite numbers. **MP2. Reason abstractly and quantitatively.** **MP7. Model with mathematics.**	Students are able to understand the meaning of a division problem.

MP8. Attend to precision. **Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.4, 6.NS.5, 6.NS.6, 6.NS.7, 6.NS.8** At this level, students use fractions, decimals and integers to represent real-world situations. They extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical which helps sixth graders move from number lines to coordinate grids. The force of this cluster is to learn about negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. This lays the foundation for working with rational numbers, algebraic expressions and equations, functions, and the coordinate plane in seventh and eight grades. **MP1. Make sense of problems and persevere in solving them.** **MP2. Reason abstractly and quantitatively.** **MP4. Model with mathematics.**	Sixth graders apply division algorithms to ivied multi-digit numbers. Students consider the reasonableness of an estimated quotient. Students relate the concepts of positive and negative numbers to real-world applications. Students attend the meaning of quantities, not just how to compute them. Students describer real-world scenario with number lines and coordinate grids and interpret the results.

K-U-D	
KNOW *Facts, formulas, information, vocabulary*	**DO** *Skills of the discipline, social skills, production skills, processes (usually verbs/verb phrases)*

• rational numbers
  ○ integers
  ○ opposites
  ○ absolute value
	• UNDERSTAND
	  ○ rational numbers as points on numbers lines
	  ○ ordered pairs as locations in coordinate plane
	  ○ absolute value as distance on number line

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absolute value as magnitude
order for rational numbers in real-world contexts

number line diagrams

relative position of two numbers on a number line diagram
distance from 0 on the number line

coordinate plane

quadrants
ordered pairs/coordinates
x axis
y axis

GCF
LCM
Perfect squares
Perfect cubes

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REPRESENT points on number lines and coordinate plane
RECOGNIZE opposites
FIND/POSITION points on number lines and coordinate planes
ORDER rational numbers
WRITE and EXPLAIN statements of order/real world context
INTERPRET relative position on number line
FIND GCF of two whole numbers less than or equal to 100
FIND LCM of two numbers less than or equal to 12
FIND the square root of perfect squares and the cubed root of perfect cubes.

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<tr>
<td colspan="2">UNDERSTAND
Big ideas, generalizations, principles, concepts, ideas that transfer across situations</td>
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Communication and collaboration with others is more efficient and accurate using rational numbers.
Negative numbers can be used to represent quantities less than zero or quantities with an associated direction.
The greatest common factor and least common multiple are used when operating with fractions.
Squares and square roots are inverses of each other. Cubes and cubed roots are inverses of each other.

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<tr>
<td colspan="2">Common Student Misconceptions for this Unit</td>
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Students might think (-6) has a greater value than (-5).
Students often confuse the order of the x and y axis.

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- Students think that it does not matter which order you follow in plotting or naming an ordered pair.
- Students do not attend to the fact that when you move left or down on a coordinate grid that you are moving in a "negative" direction, therefore, they write many of the negative coordinates as positive numbers.
- Students may confuse factors of whole numbers with multiples of whole numbers.
- Students may think that squaring a number is the same as multiplying by 2 and cubing a number is the same as multiplying by 3.
- Students may think that finding the square root is the same as dividing by 2 and finding the cubed root is the same as dividing by 3.

Unit Assessment/Performance Task	DOK
Unit 1 Part 1 and Part 2 Unit 1 Performance Task "Surface Temperature of Planets" Unit 1 Performance Task "Percent Cards"	

Vocabulary
• Absolute value • Algorithm • Common factor • Composite number • Coordinate plane • Coordinates • Cubed root • Denominator • Distributive property • Dividend

- Divisor
- Exponent
- Factor
- Factor tree
- Greatest common factor
- Inequality
- Integer
- Inverse
- Irrational number
- Multiple
- Multiplicative inverse
- Negative
- Numerator
- Opposites
- Ordered pairs
- Origin
- Perfect cube
- Perfect square
- Positive
- Prime factorization
- Prime number
- Quadrant
- Rational number
- Signed numbers
- Square root
- X-axis
- Y-axis

Key Learning Activities/Possible Lesson Focuses (order may vary)

The following activities are broken into "lessons," even though each may take more or less than one class period depending on school schedule.

Pre-assessment (Recall prior knowledge) and Pre-requisite skills review (if needed)

Lesson 1: Locations of Numbers on a Number line and Scale

Goals: 6.NS.6a

- Locate Rational numbers on a number line.
- Understand absolute value as the number of units a number is from zero.

Key Vocabulary: *integers, opposites, positive & negative numbers, absolute value*

Class will create a human number line with both negative and positive integers. Students will hold cards with a given integer and position themselves on the number line. Be sure to discuss appropriate spacing on the number line. Ask students questions about opposites and absolute value as well as the distance between two points on the number line.

Lesson 2: Prime Factorization

Lesson 3: Factors and Multiples

Students will find factors and multiples of whole numbers. Write the numbers 10, 24 and 33 on the board with two factor branches extending down from each number. Ask students what 2 numbers can be placed at the end of these branches to equal the original number. Next, place three blank lines to the right of the original numbers. Fill in one of the lines with the next multiple of each. Ask students to continue the pattern.

Goals

- Understand relationships among factors, multiples, divisors and products
- Develop strategies for finding factors and multiples, least common multiples and

greatest common factors

- Use the distributive property to express the sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor

Suggested Activities

- "I Have, Who Has…" Game
  Students participate in a whole-class activity where the first card says, "I have the first card. Who has the GCF of 8 and 12?" Then the next card would say, "I have 4. Who has the LCM of 6 and 10?" The game continues for every student to participate.
- Students can play factor/multiple games online cooperatively or individually.
  http://www.math-play.com/Factors-and-Multiples-Jeopardy/Factors-and-Multiples-Jeopardy.html
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=12
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=29

Resources

- Connected Math 2, Prime Time
- Crosswalk Coach Mathematics Grade 6, Domain 1 Lesson 1
- Teaching the Common Core Math Standards, The Number System: 6.NS.4

Lesson 4: Squares and Cubes

Lesson 5: Absolute Value and Opposites

Goals: 6. NS. 6.a 6. NS. 7

Have students work at different stations to investigate absolute value through different activities. Examples of possible stations could be: The number line station: students have to

plot as many numbers as they can that are a certain distance away from zero (2 away from 0, etc.). Students would record the appropriate answers (2 and -2, etc.) or a station with of index cards containing different positive and negative integers where they must arrange the cards from numbers with the least absolute value to the most absolute value.

Suggested Resources/Activities:

"The Number System Set 1: Integers and Absolute Value" from *Mathematics Station Activities for Common Core State Standards Grade 6*

"Finding the Opposite" *from Teaching the Common Core Math Standards with Hands-on Activities Jossey/Bass*
Tutorial on how to create a paper fortune teller: http://youtube.com/watch?v=BYarlKnetRs

Lesson 6: Using Positive and Negative Numbers to Describe Real-World Situations

Goals: 6.NS.5

Begin with a conversation about different words in real-world situations that would imply that you're dealing with a negative value vs. a positive value. (Students should name words such as: above/below, gain/loss, credit/debit, before/after, degrees below/degrees above, etc.)

Lesson 7: Quadrants on a Coordinate Grid

Goals: 6.NS.6b

Students select a place in the school that they would like to get to (example: gym, cafeteria, library). They write instructions for someone else to follow. Once they are finished, they give their instructions to someone else, and that person has to follow them and figure out where they would end up. If students had an easy time finding the correct location, ask: what words/details made the instructions easy to follow. If students did not end up at the correct location or were unable to follow the directions, ask: what could make the instructions better?

Present students with a blank coordinate grid. Ask them to make observations about it. (They should observe that it is made up of a number line and a scale, as well as the fact that it is separated into 4 sections.) If they do not make these observations, use prompting questions to help them. Once they realize it is split into 4 sections, have them mark an x anywhere they would like. Without someone else looking at their paper, have them attempt to describe to someone else how to place an x in the same exact spot on their own paper. Students will most likely be explaining up and to the right, up and to the left, down and to the right, down and to the left, etc. Ask if there is a way we could make it easier for someone to know which of the 4 sections they should be looking at in the first place? (We could all refer to them as the same names). What would be appropriate names for the different sections? (After some discussion, students should come up with 1, 2, 3, and 4). Introduce the correct language: quadrants, and model how to name them 1, 2, 3, and 4, using roman numerals: I, II, III, and IV. Students label the quadrants of their coordinate grids and again attempt to mark and x and describe to someone else how to mark an x in the same place. Discuss whether it was easier once each quadrant had a name.

Lesson 8 : Locations of Numbers on a Coordinate Plane

Goals: 6.NS.6b & 6.NS.6c

- Locate pairs of rational numbers on a coordinate plane.
- Understand signs on numbers in ordered pairs indicate locations on a coordinate plane.
- Use reflections on a coordinate plane to identify locations.

Key Vocabulary: *coordinate plane, axis, origin, quadrant, ordered pair, x-axis, y-axis*

Display a four quadrant plane on the board and a place a point at the following ordered pairs (2,3). Have students discuss various ways to arrive at the given point. This should generate discussion about why we follow the rule of x axis before y axis. Add points to quadrants 2-4 and have students explain how to arrive at those points. Once students are proficient in naming and plotting points ask questions that relate to reflections on a coordinate plane.
"If you wanted plot the same point (2, 3) in quadrant 2 what would be the new ordered pairs.

How about Q3, Q4?" Those are called reflections.

Suggested Resources/Activities:

- Coordinate Plane 2-4 & Distance in the Coordinate Plane 2-5 from *On Core Mathematics*
- Frayer Model Vocabulary Activity
- www.math-play.com/Coordinate-Plane-Jeopardy/Coordinate-Plane-Jeopardy.html
- **http://www.xpmath.com/**

Lesson 8: Distance Between Points on a Coordinate Grid

Goals: 6.NS.8

Create a grid on the floor of the room using tape OR if your floor is tiles, use the lines between the tiles as your grid. Mark the origin, x, and y axis. Have students choose a point to stand on. Ask them if anyone is on the same x coordinate as them. How can they tell? If there is someone with the same x coordinate, can you determine the distance between the two of you? Repeat this process for the y coordinates. Ask if there are any other points on the coordinate grid that are easy for them to find the distance between. *Note: students may have the misconception that you can count diagonally the same as you would count horizontally or vertically. If your students think this is the case, have them investigate this by measuring the spaces on the grid with a ruler/meter stick horizontally, vertically, then diagonally to see that the diagonal measurement is actually not the same. Have students switch places so that they are at a point where they share an x or y coordinate with another student. Ask them to answer the question: what is the distance between your two points?

Ordering of Rational Numbers using Number lines and Scales (Where does this go?) What math in focus sections?

Goals: 6.NS.7d

- Interpret statements of inequality as statements about relative position.

Key Vocabulary: *inequality*

Ask students to create a number line going from 0-10. (They could do this in notebooks or on individual whiteboards). Have students Think-Pair-Share any observations they can make about the number line. If students do not come up with the observation that the further to the left you go on the number line, the lower the numbers are (and the further to the right you go, the greater the numbers are), the teacher should ask questions to elicit this observation.

Next, ask students to create a number line going from (-10) to 10 (this can also be used as a formative assessment of the previous lesson: Locations of Numbers on a Number Line). Ask students to use the pattern they noticed about the relative location of numbers on the number line and their magnitude to decide which number on the number line is the least. They should discuss their thoughts with a partner or small group of students. Once students have realized that (-10) is the least, ask them to answer questions, such as: which numbers are greater than (-10)? Which numbers are less than (-10)? Have them write their answers as inequalities: (-10) < (-8), (-2) > (-10), etc.

Have students work with their partner to think of real-world situations for which different integers could be used (example: -10 could represent 10 degrees below 0, -52 could represent 52 feet below sea level, and 12 could represent a gain or credit of $12.00). Have them write a scenario to match one of the inequalities, stating which number is greater/less (example: (-52) < (-12) The temperature in Town A is -52 degrees Fahrenheit. The temperature in Town B is -12 degrees Fahrenheit. The temperature in Town A is less than (colder than) the temperature

in Town B).

Suggested Resources/Activities:

- Comparing and Ordering Numbers from *On Core Mathematics*

Supplemental Materials and Resources
Literature connection: Less Than Zero by Stuart J. Murphy Interdisciplinary connections: Science: positive/negative electric charge, above/below sea level, temperature Economics: credits and debits Social Studies: plotting specific locations on a map
Tools/Manipulatives
Coordinate grids Number lines Graph paper Patter blocks Two colored counters Factor trees Decimal blocks

Suggested Formative Assessment Practices/Processes
Teacher created exit slips, teacher created quizzes Think-Pair-Share: Students discuss their observations about the relative location of numbers on a number line in relation to their magnitude before sharing their ideas with the class. Agreement Circles: The teacher creates a set of statements (both true and false) related to the topic (example: when plotting a point on a coordinate grid, it does not matter which way you move first). Students stand in a circle and the teacher reads a statement. Give students a few seconds to think. If they agree with the statement, they move to the center of the circle. Fist to Five: Have students self-assess how well they understand the concept with a show of fingers (from 0-5, 0 = no understanding, up to 5, which = a complete understanding).

Differentiation and Accommodations
• Provide graphic organizers • Provide additional examples and opportunities for repetition • Provide tutoring opportunities • Provide retesting opportunities after remediation (up to teacher and district discretion) • Teach for mastery not test • Teaching concepts in different modalities • Adjust homework assignments